## Combinatorial negative curvature

Damian Osajda Problem list 6

- (1) Show that if, for an edge wz we have  $w \in H_{uv}^-$ , and  $z \in H_{uv}^+$ , then  $wz \equiv uv$ .
- (2) Show that half-spaces are convex.
- (3) Show that gated sets are convex.
- (4) Find an example of a convex set which is not gated.
- (5) Show that the intersection of gated sets is gated.
- (6) Show that the Helly property for gated sets: Any finite family of pairwise intersecting gated sets has an intersection.
- (7) Let X be a flag cubical complex such that any three squares pairwise intersecting and having a vertex in common belong to a three-cube. Show that links in X are flag.
- (8) Show that the flag cubical completion of a median graph is a  $\mathrm{CAT}(0)$  cubical complex.
- (9) Find an isometric square grid of arbitrarily large size in a cube.