Geometric and Asymptotic Group Theory II

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Blatt 3

Stalling's foldings and subgroups of free groups

(1) Show that for an S–graph Γ, the set $\overline{L(\Gamma, v)}$, of reductions of words read on Γ wrt v is a subgroup of $F(S)$. Prove that if Γ is folded, then $L(\Gamma, v)$ (unreduced words) is a subgroup of $F(S)$.

(2) Show that for an S–graph Γ its core $Core(\Gamma, v)$ wrt v satisfies the following properties:
- $Core(\Gamma, v)$ is connected and contains the vertex v;
- $Core(\Gamma, v)$ has no degree one vertices except possibly for v;
- $L(Core(\Gamma, v), v) = L(\Gamma, v)$.

(3) Show elementarily that for a subgroup H of $F(S)$ the coset S–graph $\Delta(H)$ is such that $\overline{L(\Delta(H), H)} = H$.

(4) Let Γ be a folded S–graph. Pick a spanning tree T in Γ and let T^+ be the set of all positive edges outside T. Show that $Y_T = \{[e] \mid e \in T^+\}$ is a free basis for $H = L(\Gamma, v) \leqslant F(S)$.

(5) Show that for every connected graph and for every its vertex v, there exists a geodesic wrt v spanning tree.

(6) Let Γ be a folded core S–graph, being a core graph wrt v. Pick a geodesic wrt v spanning tree T in Γ. Show that Y_T is a Nielsen-reduced free basis for $H = L(\Gamma, v)$.