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Mathematical programming and optimization

Britannica says:

> Mathematical programming: theoretical tool of management science and economics in which management operations are described by mathematical equations that can be manipulated for a variety of purposes. If the basic descriptions involved take the form of linear algebraic equations, the technique is described as linear programming.

Wiki says

> Mathematical optimization (alternatively spelled optimisation) or mathematical programming is the selection of a best element (with regard to some criterion) from some set of available alternatives. Optimization problems of sorts arise in all quantitative disciplines from computer science and engineering to operations research and economics, and the development of solution methods has been of interest in mathematics for centuries.

Mathematical programming is the art of finding sensible algorithms solving concrete problems in practice.

Optimization problem ask to find an element in a given set that is best possible, from some perspective.

Typically, we want to find the extrema of a given function

$$f : A \to \mathbb{R}.$$

In mathematics we prove

Theorem. *If A is a closed bounded subset of $\mathbb{R}^n$ and $f : A \to \mathbb{R}$ is continuous then there is $x_0 \in A$ at which f attains its minimum.*

To do mathematical programming you are to design a method of finding that x_0.

This requires some concrete nontrivial ideas ever if A is a polyhedron (is defined by a finite number of linear inequalities and equations) and f is a linear function

$\Longrightarrow$ **Linear programming**.

If the set A is finite then you might try to compare all the values of f but even in simple cases the set is to big to do it in a limited time

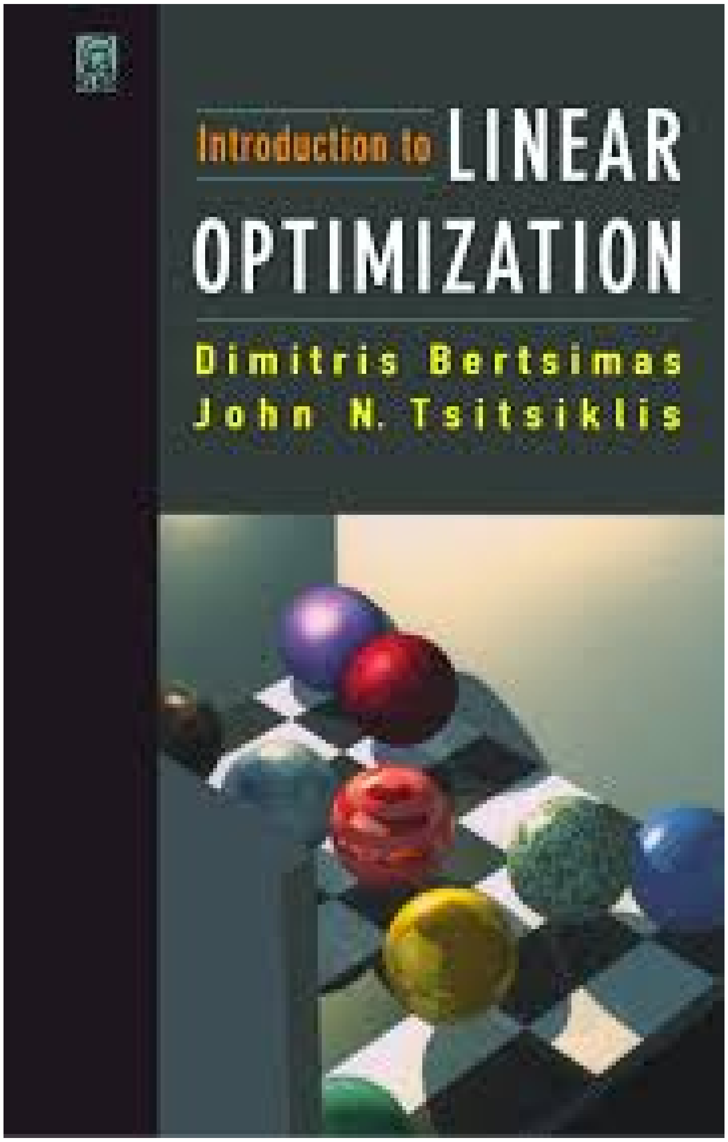
$\Longrightarrow$ **Discrete programming**.

Example. *Find the shortest path between two vertices of a given graph in which every edge has a given length.*

The knapsack problem. You are smuggling some goods across the border. You have n items, the ith item is of weight w_i and value of p_i. The capacity of you knapsack is C. How to pack it so that the total value is as large as possible?

The formulation: Maximize $\Sigma_{i=1}^{n} p_i x_i$ subject to

$$\sum_{i=1}^{n} w_i x_i \leqslant C, \quad x_i \in \{0, 1\}.$$


Introduction to LINEAR OPTIMIZATION
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