

Kolokwium nr 6: poniedziałek 25.11.2019, godz. 10:15-11:00, materiał zad. 1–421.

Poziom standardowy (z myślą o ocenie co najwyżej dobrej)

6. Kresy zbiorów.

Zadania do omówienia na ćwiczeniach 18,21.11.2019 (grupy 1, 2, 4).

Zadania należy spróbować rozwiązać przed ćwiczeniami.

**Pomoc w rozwiązaniu tych zadań można uzyskać także
na ćwiczeniach grupy 5 w czwartek 21.11.2019.**

W każdym z poniższych zadań podaj kresy zbioru Z oraz określ, czy kresy należą do zbioru Z . Nie wszystkie zadania będą omówione szczegółowo na ćwiczeniach – studenci powinni umieć wskazać zadania, które sprawiły największą trudność.

277. $Z = \left\{ \frac{1}{n^2 - 7} : n \in \mathbb{N} \right\}$

278. $Z = \left\{ x^n : x \in \left(-\frac{1}{2}, \frac{1}{5} \right) \wedge n \in \mathbb{N} \right\}$

279. $Z = \left\{ \sqrt{n^2 + 3} - n : n \in \mathbb{N} \right\}$

280. $Z = \{ \log_2(2n - 1) - \log_2 n : n \in \mathbb{N} \}$

281. $Z = \left\{ \frac{n}{3n + 7} : n \in \mathbb{N} \right\}$

282. $Z = \left\{ \frac{n}{3n - 7} : n \in \mathbb{N} \right\}$

283. $Z = \left\{ \frac{(\log_2(n^2 + 1)) \cdot \log_3(n^2 + 4)}{(\log_8(n^2 + 4)) \cdot \log_9(n^2 + 1)} : n \in \mathbb{N} \right\}$

284. $Z = \left\{ \sqrt{x^2 + 2x + 1} : -5 \leq x < 3 \right\}$

285. $Z = \left\{ \frac{1}{x^2 + 2} : x \in \mathbb{R} \right\}$

286. $Z = \left\{ \frac{x^2 + 1}{x^2 + 2} : x \in \mathbb{R} \right\}$

287. $Z = \{ x^2 + 4x + 4 : x \in (-6, 1) \}$

288. $Z = \{ x^2 + 4y + 4 : x, y \in (-6, 1) \}$

289. $Z = \left\{ \frac{2n^2 + 3n + 5}{2n^2 + 3n + 4} : n \in \mathbb{N} \right\}$

$$290. Z = \left\{ \frac{2n^2 + 3n + 5}{2n^2 + 3n + 6} : n \in \mathbb{N} \right\}$$

$$291. Z = \{x^2 : x \in (-3, 2)\}$$

$$292. Z = \{x^3 : x \in (-3, 2)\}$$

$$293. Z = \left\{ \frac{1}{5n - 13} : n \in \mathbb{N} \right\}$$

$$294. Z = \left\{ \frac{\sqrt[n]{2}}{n} : n \in \mathbb{N} \right\}$$

$$295. Z = \{n^2 - 5n : n \in \mathbb{N}\}$$

$$296. Z = \left\{ \frac{1}{n+1} - \frac{1}{m+2} : m, n \in \mathbb{N} \right\}$$

$$297. Z = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \wedge 2m^2 < 3n^2 \right\}$$

$$298. Z = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \wedge 2^m > 3^n \right\}$$

$$299. Z = \left\{ 7n + \frac{n! + n^{2018} + 1}{n! + n^{2018} + 4} : n \in \mathbb{N} \right\}$$

$$300. Z = \{x^2 : x \in (-4, 9)\}$$

$$301. Z = \left\{ \frac{n}{2n+3} : n \in \mathbb{N} \right\}$$

$$302. Z = \left\{ \binom{2018}{n} : n \in \mathbb{N} \wedge n \leq 2018 \right\}$$

$$303. Z = \left\{ \frac{n}{n+m} : m, n \in \mathbb{N} \right\}$$

$$304. Z = \left\{ \left(\frac{1}{n} - \frac{2}{3} \right)^2 : n \in \mathbb{N} \right\}$$

$$305. Z = \{ \sqrt{n^2 + 2n} - n : n \in \mathbb{N} \}$$

$$306. Z = \{ \sqrt[n]{3} - \sqrt[m]{2} : m, n \in \mathbb{N} \}$$

$$307. Z = \left\{ \frac{7}{n} - 3m : m, n \in \mathbb{N} \right\}$$

$$308. Z = \left\{ \left(\sqrt{37} - 5 \right)^n : n \in \mathbb{N} \right\}$$

$$309. Z = \left\{ \left(\sqrt{37} - 6 \right)^n : n \in \mathbb{N} \right\}$$

$$310. Z = \left\{ \left(\sqrt{37} - 7 \right)^n : n \in \mathbb{N} \right\}$$

$$311. Z = \left\{ \left(\sqrt{37} - 8 \right)^n : n \in \mathbb{N} \right\}$$

$$312. Z = \left\{ \frac{5m - 2n}{mn} : m, n \in \mathbb{N} \right\}$$

$$313. Z = \left\{ \frac{m}{n+7} : m, n \in \mathbb{N} \right\}$$

$$314. Z = \left\{ x^2 : x \in (-2, 1) \right\}$$

$$315. Z = \left\{ x^3 : x \in (-2, 1) \right\}$$

$$316. Z = \left\{ 3x^2 + y^3 : x, y \in (-2, 1) \right\}$$

$$317. Z = \left\{ |2 - \log_2 x| : x \in (1, 8] \right\}$$

$$318. Z = \left\{ |2 - \log_2 x| : x \in (1, 16] \right\}$$

$$319. Z = \left\{ |2 - \log_2 x| : x \in (1, 32] \right\}$$

$$320. Z = \left\{ \frac{1}{3n-2} + \frac{1}{2m-3} : m, n \in \mathbb{N} \right\}$$

$$321. Z = \left\{ \log_2(n+7) - \log_2 n : n \in \mathbb{N} \right\}$$

$$322. Z = \left\{ \frac{m+n}{\sqrt{mn}} : m, n \in \mathbb{N} \right\}$$

$$323. Z = \left\{ \frac{(-1)^n}{n^2+1} : n \in \mathbb{N} \right\}$$

$$324. Z = \left\{ \frac{1}{n^2-22} : n \in \mathbb{N} \right\}$$

$$325. Z = \left\{ \frac{2n+1}{3n+1} : n \in \mathbb{N} \right\}$$

$$326. Z = \left\{ \frac{2n+1}{3n+2} : n \in \mathbb{N} \right\}$$

$$327. Z = \{x - 2y : x, y \in \mathbb{R} \wedge 16 < x \leq 28 \wedge 3 < y \leq 4\}$$

$$328. Z = \{|x - y| : x, y \in \mathbb{R} \wedge 16 < x \leq 28 \wedge 3 < y \leq 4\}$$

$$329. Z = \left\{ \frac{1}{7n-30} : n \in \mathbb{N} \right\}$$

$$330. Z = \left\{ \frac{1}{(7n-30)^2} : n \in \mathbb{N} \right\}$$

$$331. Z = \left\{ \frac{1}{(7n-30)^3} : n \in \mathbb{N} \right\}$$

$$332. Z = \left\{ \frac{1}{7m-30} + \frac{1}{(7n-30)^2} : m, n \in \mathbb{N} \right\}$$

$$333. Z = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \wedge 5^3 \cdot n^{15} \leq m^{15} \leq 3^5 \cdot n^{15} \right\}$$

$$334. Z = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \wedge 5^2 \cdot n^{10} \leq m^{10} \leq 2^5 \cdot n^{10} \right\}$$

$$335. Z = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \wedge 3^2 \cdot n^6 \leq m^6 \leq 2^3 \cdot n^6 \right\}$$

$$336. Z = \left\{ \frac{1}{n+2} : n \in \mathbb{N} \right\}$$

$$337. Z = \left\{ \frac{2n+5}{n+2} : n \in \mathbb{N} \right\}$$

$$338. Z = \left\{ \frac{2n+3}{n+2} : n \in \mathbb{N} \right\}$$

$$339. Z = \left\{ \frac{m+n}{mn} : m, n \in \mathbb{N} \right\}$$

$$340. Z = \left\{ \frac{8m-3n}{mn} : m, n \in \mathbb{N} \right\}$$

$$341. Z = \left\{ \frac{m+2n+3}{mn} : m, n \in \mathbb{N} \right\}$$

$$342. Z = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \wedge 25n^2 \leq m^2 \leq 27n^2 \right\}$$

$$343. Z = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \wedge 25n^3 \leq m^3 \leq 27n^3 \right\}$$

$$344. Z = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \wedge 25^n \leq 3^m \leq 27^n \right\}$$

$$345. Z = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \wedge 25^n \leq 5^m \leq 27^n \right\}$$

$$346. Z = \{x^2 : x \in (-3, 1)\}$$

$$347. Z = \{x^3 : x \in (-3, 1)\}$$

$$348. Z = \{x^4 : x \in (-3, 1)\}$$

$$349. Z = \{x^2 - 2x + 1 : x \in (-1, 4)\}$$

$$350. Z = \{x^2 - 4x + 4 : x \in (-1, 4)\}$$

$$351. Z = \{x^2 - 6x + 9 : x \in (-1, 4)\}$$

$$352. Z = \{x^2 - 2x : x \in (-1, 4)\}$$

$$353. Z = \{x^2 - 4x : x \in (-1, 4)\}$$

$$354. Z = \{x^2 - 6x : x \in (-1, 4)\}$$

$$355. Z = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \wedge 16n^2 \leq m^2 \leq 64n^2 \right\}$$

$$356. Z = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \wedge 16n^3 \leq m^3 \leq 64n^3 \right\}$$

$$357. Z = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \wedge 16n^4 \leq m^4 \leq 64n^4 \right\}$$

$$358. Z = \{\sqrt{n^4 + n^2} - n^2 : n \in \mathbb{N}\}$$

$$359. Z = \{\sqrt[4]{n^4 + n^3} - n : n \in \mathbb{N}\}$$

$$360. Z = \left\{ (\log_2 x)^2 : x \in \left(\frac{1}{8}, 2 \right) \right\}$$

$$361. Z = \left\{ (\log_3 x)^3 : x \in \left(\frac{1}{9}, 3 \right) \right\}$$

$$362. Z = \left\{ (\log_4 x)^4 : x \in \left(\frac{1}{16}, 4 \right) \right\}$$

$$363. Z = \left\{ \log_x 8 : x \in \left(0, \frac{1}{2} \right] \right\}$$

$$364. Z = \left\{ \log_x 8 : x \in [\sqrt{2}, +\infty) \right\}$$

$$365. Z = \{\log_x 8 : x \in (1, 4]\}$$

$$366. Z = \left\{ \log_x 8 : x \in \left[\frac{1}{16}, 1 \right) \right\}$$

$$367. Z = \{(x-2)^2 : x \in (0, 3)\}$$

$$368. Z = \{(x-2)^3 : x \in (0, 3)\}$$

$$369. Z = \{(x-2)^4 : x \in (0, 3)\}$$

$$370. Z = \{(x-2)^5 : x \in (0, 3)\}$$

$$371. Z = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \wedge 4n^2 \leq 8m^2 \leq 16n^2 \right\}$$

$$372. Z = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \wedge 4n^2 \leq 9m^2 \leq 16n^2 \right\}$$

$$373. Z = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \wedge 25n^2 \leq 9m^2 \leq 27n^2 \right\}$$

$$374. Z = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \wedge 4^n \leq 8^m \leq 16^n \right\}$$

$$375. Z = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \wedge 4^n \leq 9^m \leq 16^n \right\}$$

$$376. Z = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \wedge 25^n \leq 9^m \leq 27^n \right\}$$

$$377. Z = \left\{ \frac{1}{3^n - 10} : n \in \mathbb{N} \right\}$$

$$378. Z = \left\{ \frac{1}{3^n - 20} : n \in \mathbb{N} \right\}$$

$$379. Z = \left\{ \frac{1}{3^n - 26} : n \in \mathbb{N} \right\}$$

$$380. Z = \left\{ \frac{1}{5^n - 26} : n \in \mathbb{N} \right\}$$

$$381. Z = \left\{ \left(\sqrt{26} - 4 \right)^n : n \in \mathbb{N} \right\}$$

$$382. Z = \left\{ \left(\sqrt{26} - 5 \right)^n : n \in \mathbb{N} \right\}$$

$$383. Z = \left\{ \left(\sqrt{26} - 6 \right)^n : n \in \mathbb{N} \right\}$$

$$384. Z = \{2^{x^2} : x \in (-2, 1)\}$$

$$385. Z = \{2^{x^3} : x \in (-2, 1)\}$$

$$386. Z = \{2^{x^4} : x \in (-2, 1)\}$$

$$387. Z = \left\{ \frac{1}{n^2 - 44} : n \in \mathbb{N} \right\}$$

$$388. Z = \left\{ \frac{(-1)^n}{n^2 + 44} : n \in \mathbb{N} \right\}$$

$$389. Z = \left\{ \frac{(-1)^n}{n^2 - 44} : n \in \mathbb{N} \right\}$$

$$390. Z = \left\{ \left(\frac{-1}{3} \right)^n : n \in \mathbb{N} \right\}$$

$$391. Z = \left\{ \sum_{i=1}^n \frac{1}{3^i} : n \in \mathbb{N} \right\}$$

$$392. Z = \left\{ \frac{1}{n^2 - 40n + 350} : n \in \mathbb{N} \right\}$$

$$393. Z = \left\{ \frac{1}{n^2 - 40n + 370} : n \in \mathbb{N} \right\}$$

$$394. Z = \left\{ \frac{1}{n^2 - 40n + 390} : n \in \mathbb{N} \right\}$$

$$395. Z = \left\{ \frac{1}{n^2 - 40n + 410} : n \in \mathbb{N} \right\}$$

$$396. Z = \left\{ \frac{1}{n^2 - 40n + 430} : n \in \mathbb{N} \right\}$$

$$397. Z = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \wedge 3^{n^2} \leq 2^{m^2} \leq 5^{n^2} \right\}$$

$$398. Z = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \wedge 4^{n^2} \leq 2^{m^2} \leq 8^{n^2} \right\}$$

$$399. Z = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \wedge 9^{n^2} \leq 3^{m^2} \leq 27^{n^2} \right\}$$

$$400. Z = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \wedge 16^{n^2} \leq 2^{m^2} \leq 32^{n^2} \right\}$$

$$401. Z = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \wedge 27^{n^2} \leq 3^{m^2} \leq 81^{n^2} \right\}$$

Niech $\mathbb{T}$ będzie zbiorem wszystkich ciągów (a_n) spełniających warunek

$$\forall_{n \in \mathbb{N}} |a_n - 1| < \frac{1}{n}.$$

W każdym z dziesięciu poniższych zadań podaj odpowiedni kres zbioru.

402. $\sup\{a_1 : (a_n) \in \mathbb{T}\} = \dots\dots\dots$

403. $\inf\{a_1 : (a_n) \in \mathbb{T}\} = \dots\dots\dots$

404. $\sup\{a_2 : (a_n) \in \mathbb{T}\} = \dots\dots\dots$

405. $\inf\{a_2 : (a_n) \in \mathbb{T}\} = \dots\dots\dots$

406. $\sup\{a_2 - a_3 : (a_n) \in \mathbb{T}\} = \dots\dots\dots$

407. $\inf\{a_2 - a_3 : (a_n) \in \mathbb{T}\} = \dots\dots\dots$

408. $\sup\{a_3 - a_6 : (a_n) \in \mathbb{T}\} = \dots\dots\dots$

409. $\inf\{a_3 - a_6 : (a_n) \in \mathbb{T}\} = \dots\dots\dots$

410. $\sup\{a_2 + a_3 + a_6 : (a_n) \in \mathbb{T}\} = \dots\dots\dots$

411. $\inf\{a_2 + a_3 + a_6 : (a_n) \in \mathbb{T}\} = \dots\dots\dots$

Niech $\mathbb{T}$ będzie zbiorem wszystkich ciągów (a_n) spełniających warunek

$$\forall_{n \in \mathbb{N}} \left| a_n - \frac{1}{n} \right| < \frac{1}{n}.$$

W każdym z dziesięciu poniższych zadań podaj odpowiedni kres zbioru.

412. $\sup\{a_1 : (a_n) \in \mathbb{T}\} = \dots\dots\dots$

413. $\inf\{a_1 : (a_n) \in \mathbb{T}\} = \dots\dots\dots$

414. $\sup\{a_2 : (a_n) \in \mathbb{T}\} = \dots\dots\dots$

415. $\inf\{a_2 : (a_n) \in \mathbb{T}\} = \dots\dots\dots$

416. $\sup\{a_2 - a_3 : (a_n) \in \mathbb{T}\} = \dots\dots\dots$

417. $\inf\{a_2 - a_3 : (a_n) \in \mathbb{T}\} = \dots\dots\dots$

418. $\sup\{a_3 - a_6 : (a_n) \in \mathbb{T}\} = \dots\dots\dots$

419. $\inf\{a_3 - a_6 : (a_n) \in \mathbb{T}\} = \dots\dots\dots$

420. $\sup\{a_2 + a_3 + a_6 : (a_n) \in \mathbb{T}\} = \dots\dots\dots$

421. $\inf\{a_2 + a_3 + a_6 : (a_n) \in \mathbb{T}\} = \dots\dots\dots$