

ANALIZA 2

20 marca 2026 r., godz. 8:15–9:45

Wykładowca: Jarosław Wróblewski

W każdym zadaniu za 0, 1, 2, 3, 4 poprawne odpowiedzi otrzymuje się odpowiednio 0, 1, 3, 6, 10 punktów.

Zadania **11 i 12** to zadania dodatkowe (niezbyt trudne, ale wymagające sprawnego zapanowania nad rachunkami).

Podczas testu nie wolno korzystać z kalkulatorów.

Odpowiedzi należy podawać w postaci uproszczonej.

Pisz czytelnie !!!

$$1. \text{ a) } \int x \cdot (2x^2 + 1)^4 dx = \frac{(2x^2 + 1)^5}{20} \quad \text{b) } \int x \cdot (3x^2 + 2)^5 dx = \frac{(3x^2 + 2)^6}{36}$$

$$\text{c) } \int x \cdot (4x^2 + 3)^6 dx = \frac{(4x^2 + 3)^7}{56} \quad \text{d) } \int x \cdot (5x^2 + 4)^7 dx = \frac{(5x^2 + 4)^8}{80}$$

$$2. \text{ a) } \int \frac{x^5 dx}{(x^6 + 1)^5} = \frac{-1}{24 \cdot (x^6 + 1)^4} \quad \text{b) } \int \frac{x^4 dx}{(x^5 + 1)^4} = \frac{-1}{15 \cdot (x^5 + 1)^3}$$

$$\text{c) } \int \frac{x^3 dx}{(x^4 + 1)^3} = \frac{-1}{8 \cdot (x^4 + 1)^2} \quad \text{d) } \int \frac{x^2 dx}{(x^3 + 1)^2} = \frac{-1}{3 \cdot (x^3 + 1)}$$

$$3. \text{ a) } \int \frac{x^4 dx}{x^5 + 1} = \frac{\ln|x^5 + 1|}{5}$$

$$\text{b) } \int \frac{x^9 dx}{x^{10} + 1} = \frac{\ln(x^{10} + 1)}{10}$$

$$\text{c) } \int \frac{x^{19} dx}{x^{20} + 1} = \frac{\ln(x^{20} + 1)}{20}$$

$$\text{d) } \int \frac{x^{39} dx}{x^{40} + 1} = \frac{\ln(x^{40} + 1)}{40}$$

$$4. \text{ a) } \int \frac{x^{39} dx}{x^{80} + 1} = \frac{\arctg(x^{40})}{40}$$

$$\text{b) } \int \frac{x^{19} dx}{x^{40} + 1} = \frac{\arctg(x^{20})}{20}$$

$$\text{c) } \int \frac{x^9 dx}{x^{20} + 1} = \frac{\arctg(x^{10})}{10}$$

$$\text{d) } \int \frac{x^4 dx}{x^{10} + 1} = \frac{\arctg(x^5)}{5}$$

$$5. \text{ a) } \int x^2 \cdot \sqrt{x^3 + 1} dx = \frac{2 \cdot (x^3 + 1)^{3/2}}{9}$$

$$\text{b) } \int x \cdot \sqrt{x^2 + 1} dx = \frac{(x^2 + 1)^{3/2}}{3}$$

$$\text{c) } \int x^4 \cdot \sqrt{x^5 + 1} dx = \frac{2 \cdot (x^5 + 1)^{3/2}}{15}$$

$$\text{d) } \int x^3 \cdot \sqrt{x^4 + 1} dx = \frac{(x^4 + 1)^{3/2}}{6}$$

$$6. \text{ a) } \int \frac{x^4 dx}{\sqrt[8]{x^5 + 1}} = \frac{8 \cdot (x^5 + 1)^{7/8}}{35}$$

$$\text{b) } \int \frac{x dx}{\sqrt{x^2 + 1}} = \sqrt{x^2 + 1}$$

$$\text{c) } \int \frac{x^2 dx}{\sqrt[4]{x^3 + 1}} = \frac{4 \cdot (x^3 + 1)^{3/4}}{9}$$

$$\text{d) } \int \frac{x^3 dx}{\sqrt[6]{x^4 + 1}} = \frac{3 \cdot (x^4 + 1)^{5/6}}{10}$$

$$7. \text{ a) } \int \sqrt[3]{x^5 + x^3} dx = \frac{3 \cdot (x^2 + 1)^{4/3}}{8}$$

$$\text{b) } \int \sqrt[3]{x^9 + x^6} dx = \frac{(x^3 + 1)^{4/3}}{4}$$

$$\text{c) } \int \sqrt[3]{x^{13} + x^9} dx = \frac{3 \cdot (x^4 + 1)^{4/3}}{16}$$

$$\text{d) } \int \sqrt[3]{x^{17} + x^{12}} dx = \frac{3 \cdot (x^5 + 1)^{4/3}}{20}$$

$$8. \text{ a) } \int \sqrt[5]{x^{25} + x^{20}} dx = \frac{(x^5 + 1)^{6/5}}{6}$$

$$\text{b) } \int \sqrt[5]{x^{19} + x^{15}} dx = \frac{5 \cdot (x^4 + 1)^{6/5}}{24}$$

$$\text{c) } \int \sqrt[5]{x^{13} + x^{10}} dx = \frac{5 \cdot (x^3 + 1)^{6/5}}{18}$$

$$\text{d) } \int \sqrt[5]{x^7 + x^5} dx = \frac{5 \cdot (x^2 + 1)^{6/5}}{12}$$

$$9. \text{ a) } \int \frac{(\ln x)^2 dx}{x} = \frac{(\ln x)^3}{3}$$

$$\text{b) } \int \frac{\ln x dx}{x} = \frac{(\ln x)^2}{2}$$

$$\text{c) } \int \frac{(\operatorname{arctg} x)^2 dx}{x^2 + 1} = \frac{(\operatorname{arctg} x)^3}{3}$$

$$\text{d) } \int \frac{\operatorname{arctg} x dx}{x^2 + 1} = \frac{(\operatorname{arctg} x)^2}{2}$$

$$10. \text{ a) } \int \sqrt[3]{x} \cdot \ln x dx = \frac{3 \cdot x^{4/3} \cdot \ln x}{4} - \frac{9 \cdot x^{4/3}}{16}$$

$$\text{b) } \int x^2 \cdot \ln x dx = \frac{x^3 \cdot \ln x}{3} - \frac{x^3}{9}$$

$$\text{c) } \int x^3 \cdot \ln x dx = \frac{x^4 \cdot \ln x}{4} - \frac{x^4}{16}$$

$$\text{d) } \int \sqrt{x} \cdot \ln x dx = \frac{2 \cdot x^{3/2} \cdot \ln x}{3} - \frac{4 \cdot x^{3/2}}{9}$$

$$\begin{aligned}
 11. \text{ a) } \int x \cdot \sqrt{x+1} \, dx &= \frac{2 \cdot (x+1)^{5/2}}{5} - \frac{2 \cdot (x+1)^{3/2}}{3} = \\
 &= \frac{2 \cdot x \cdot (x+1)^{3/2}}{3} - \frac{4 \cdot (x+1)^{5/2}}{15}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } \int x \cdot \sqrt[3]{x+1} \, dx &= \frac{3 \cdot (x+1)^{7/3}}{7} - \frac{3 \cdot (x+1)^{4/3}}{4} = \\
 &= \frac{3 \cdot x \cdot (x+1)^{4/3}}{4} - \frac{9 \cdot (x+1)^{7/3}}{28}
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 \text{c) } \int x \cdot \sqrt[4]{x+1} \, dx &= \frac{4 \cdot (x+1)^{9/4}}{9} - \frac{4 \cdot (x+1)^{5/4}}{5} = \\
 &= \frac{4 \cdot x \cdot (x+1)^{5/4}}{5} - \frac{16 \cdot (x+1)^{9/4}}{45}
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 \text{d) } \int x \cdot \sqrt[5]{x+1} \, dx &= \frac{5 \cdot (x+1)^{11/5}}{11} - \frac{5 \cdot (x+1)^{6/5}}{6} = \\
 &= \frac{5 \cdot x \cdot (x+1)^{6/5}}{6} - \frac{25 \cdot (x+1)^{11/5}}{66}
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$$\begin{aligned}
 12. \text{ a) } \int \frac{x \, dx}{\sqrt[5]{x+1}} &= \frac{5 \cdot (x+1)^{9/5}}{9} - \frac{5 \cdot (x+1)^{4/5}}{4} = \\
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$$\begin{aligned}
 \text{d) } \int \frac{x \, dx}{\sqrt{x+1}} &= \frac{2 \cdot (x+1)^{3/2}}{3} - 2 \cdot \sqrt{x+1} = 2 \cdot x \cdot \sqrt{x+1} - \frac{4 \cdot (x+1)^{3/2}}{3}
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$$3. \text{ a) } \int \frac{x^{19} dx}{x^{20} + 1} = \frac{\ln(x^{20} + 1)}{20}$$

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 \end{aligned}$$

$$\begin{aligned}
 \text{b) } \int \frac{x dx}{\sqrt[3]{x+1}} &= \frac{3 \cdot (x+1)^{5/3}}{5} - \frac{3 \cdot (x+1)^{2/3}}{2} = \\
 &= \frac{3 \cdot x \cdot (x+1)^{2/3}}{2} - \frac{9 \cdot (x+1)^{5/3}}{10}
 \end{aligned}$$

$$\text{c) } \int \frac{x dx}{\sqrt{x+1}} = \frac{2 \cdot (x+1)^{3/2}}{3} - 2 \cdot \sqrt{x+1} = 2 \cdot x \cdot \sqrt{x+1} - \frac{4 \cdot (x+1)^{3/2}}{3}$$

$$\begin{aligned}
 \text{d) } \int \frac{x dx}{\sqrt[4]{x+1}} &= \frac{4 \cdot (x+1)^{7/4}}{7} - \frac{4 \cdot (x+1)^{3/4}}{3} = \\
 &= \frac{4 \cdot x \cdot (x+1)^{3/4}}{3} - \frac{16 \cdot (x+1)^{7/4}}{21}
 \end{aligned}$$

ANALIZA 2

20 marca 2026 r., godz. 8:15–9:45

Wykładowca: Jarosław Wróblewski

W każdym zadaniu za 0, 1, 2, 3, 4 poprawne odpowiedzi otrzymuje się odpowiednio 0, 1, 3, 6, 10 punktów.

Zadania **11 i 12** to zadania dodatkowe (niezbyt trudne, ale wymagające sprawnego zapanowania nad rachunkami).

Podczas testu nie wolno korzystać z kalkulatorów.

Odpowiedzi należy podawać w postaci uproszczonej.

Pisz czytelnie !!!

$$1. \text{ a) } \int x \cdot (2x^2 + 1)^4 dx = \frac{(2x^2 + 1)^5}{20} \quad \text{b) } \int x \cdot (5x^2 + 4)^7 dx = \frac{(5x^2 + 4)^8}{80}$$

$$\text{c) } \int x \cdot (3x^2 + 2)^5 dx = \frac{(3x^2 + 2)^6}{36} \quad \text{d) } \int x \cdot (4x^2 + 3)^6 dx = \frac{(4x^2 + 3)^7}{56}$$

$$2. \text{ a) } \int \frac{x^5 dx}{(x^6 + 1)^5} = \frac{-1}{24 \cdot (x^6 + 1)^4} \quad \text{b) } \int \frac{x^2 dx}{(x^3 + 1)^2} = \frac{-1}{3 \cdot (x^3 + 1)}$$

$$\text{c) } \int \frac{x^4 dx}{(x^5 + 1)^4} = \frac{-1}{15 \cdot (x^5 + 1)^3} \quad \text{d) } \int \frac{x^3 dx}{(x^4 + 1)^3} = \frac{-1}{8 \cdot (x^4 + 1)^2}$$

$$3. \text{ a) } \int \frac{x^4 dx}{x^5 + 1} = \frac{\ln|\mathbf{x}^5 + 1|}{5}$$

$$\text{b) } \int \frac{x^{39} dx}{x^{40} + 1} = \frac{\ln(\mathbf{x}^{40} + 1)}{40}$$

$$\text{c) } \int \frac{x^9 dx}{x^{10} + 1} = \frac{\ln(\mathbf{x}^{10} + 1)}{10}$$

$$\text{d) } \int \frac{x^{19} dx}{x^{20} + 1} = \frac{\ln(\mathbf{x}^{20} + 1)}{20}$$

$$4. \text{ a) } \int \frac{x^{39} dx}{x^{80} + 1} = \frac{\arctg(\mathbf{x}^{40})}{40}$$

$$\text{b) } \int \frac{x^4 dx}{x^{10} + 1} = \frac{\arctg(\mathbf{x}^5)}{5}$$

$$\text{c) } \int \frac{x^{19} dx}{x^{40} + 1} = \frac{\arctg(\mathbf{x}^{20})}{20}$$

$$\text{d) } \int \frac{x^9 dx}{x^{20} + 1} = \frac{\arctg(\mathbf{x}^{10})}{10}$$

$$5. \text{ a) } \int x^4 \cdot \sqrt{x^5 + 1} dx = \frac{2 \cdot (\mathbf{x}^5 + 1)^{3/2}}{15}$$

b)

$$\int x^2 \cdot \sqrt{x^3 + 1} dx = \frac{2 \cdot (\mathbf{x}^3 + 1)^{3/2}}{9}$$

$$\text{c) } \int x^3 \cdot \sqrt{x^4 + 1} dx = \frac{(\mathbf{x}^4 + 1)^{3/2}}{6}$$

$$\text{d) } \int x \cdot \sqrt{x^2 + 1} dx = \frac{(\mathbf{x}^2 + 1)^{3/2}}{3}$$

$$6. \text{ a) } \int \frac{x^4 dx}{\sqrt[8]{x^5 + 1}} = \frac{8 \cdot (\mathbf{x}^5 + 1)^{7/8}}{35}$$

$$\text{b) } \int \frac{x^3 dx}{\sqrt[6]{x^4 + 1}} = \frac{3 \cdot (\mathbf{x}^4 + 1)^{5/6}}{10}$$

$$\text{c) } \int \frac{x dx}{\sqrt{x^2 + 1}} = \sqrt{\mathbf{x}^2 + 1}$$

$$\text{d) } \int \frac{x^2 dx}{\sqrt[4]{x^3 + 1}} = \frac{4 \cdot (\mathbf{x}^3 + 1)^{3/4}}{9}$$

$$7. \text{ a) } \int \sqrt[3]{x^5 + x^3} dx = \frac{3 \cdot (x^2 + 1)^{4/3}}{8}$$

$$\text{b) } \int \sqrt[3]{x^{17} + x^{12}} dx = \frac{3 \cdot (x^5 + 1)^{4/3}}{20}$$

$$\text{c) } \int \sqrt[3]{x^9 + x^6} dx = \frac{(x^3 + 1)^{4/3}}{4}$$

$$\text{d) } \int \sqrt[3]{x^{13} + x^9} dx = \frac{3 \cdot (x^4 + 1)^{4/3}}{16}$$

$$8. \text{ a) } \int \sqrt[5]{x^{25} + x^{20}} dx = \frac{(x^5 + 1)^{6/5}}{6}$$

$$\text{b) } \int \sqrt[5]{x^7 + x^5} dx = \frac{5 \cdot (x^2 + 1)^{6/5}}{12}$$

$$\text{c) } \int \sqrt[5]{x^{19} + x^{15}} dx = \frac{5 \cdot (x^4 + 1)^{6/5}}{24}$$

$$\text{d) } \int \sqrt[5]{x^{13} + x^{10}} dx = \frac{5 \cdot (x^3 + 1)^{6/5}}{18}$$

$$9. \text{ a) } \int \frac{(\arctg x)^2 dx}{x^2 + 1} = \frac{(\arctg x)^3}{3}$$

$$\text{b) } \int \frac{(\ln x)^2 dx}{x} = \frac{(\ln x)^3}{3}$$

$$\text{c) } \int \frac{\arctg x dx}{x^2 + 1} = \frac{(\arctg x)^2}{2}$$

$$\text{d) } \int \frac{\ln x dx}{x} = \frac{(\ln x)^2}{2}$$

$$10. \text{ a) } \int \sqrt[3]{x} \cdot \ln x dx = \frac{3 \cdot x^{4/3} \cdot \ln x}{4} - \frac{9 \cdot x^{4/3}}{16}$$

$$\text{b) } \int \sqrt{x} \cdot \ln x dx = \frac{2 \cdot x^{3/2} \cdot \ln x}{3} - \frac{4 \cdot x^{3/2}}{9}$$

$$\text{c) } \int x^2 \cdot \ln x dx = \frac{x^3 \cdot \ln x}{3} - \frac{x^3}{9}$$

$$\text{d) } \int x^3 \cdot \ln x dx = \frac{x^4 \cdot \ln x}{4} - \frac{x^4}{16}$$

$$\begin{aligned}
 11. \text{ a) } \int x \cdot \sqrt{x+1} dx &= \frac{2 \cdot (x+1)^{5/2}}{5} - \frac{2 \cdot (x+1)^{3/2}}{3} = \\
 &= \frac{2 \cdot x \cdot (x+1)^{3/2}}{3} - \frac{4 \cdot (x+1)^{5/2}}{15}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } \int x \cdot \sqrt[5]{x+1} dx &= \frac{5 \cdot (x+1)^{11/5}}{11} - \frac{5 \cdot (x+1)^{6/5}}{6} = \\
 &= \frac{5 \cdot x \cdot (x+1)^{6/5}}{6} - \frac{25 \cdot (x+1)^{11/5}}{66}
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } \int x \cdot \sqrt[3]{x+1} dx &= \frac{3 \cdot (x+1)^{7/3}}{7} - \frac{3 \cdot (x+1)^{4/3}}{4} = \\
 &= \frac{3 \cdot x \cdot (x+1)^{4/3}}{4} - \frac{9 \cdot (x+1)^{7/3}}{28}
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } \int x \cdot \sqrt[4]{x+1} dx &= \frac{4 \cdot (x+1)^{9/4}}{9} - \frac{4 \cdot (x+1)^{5/4}}{5} = \\
 &= \frac{4 \cdot x \cdot (x+1)^{5/4}}{5} - \frac{16 \cdot (x+1)^{9/4}}{45}
 \end{aligned}$$

$$\begin{aligned}
 12. \text{ a) } \int \frac{x dx}{\sqrt[5]{x+1}} &= \frac{5 \cdot (x+1)^{9/5}}{9} - \frac{5 \cdot (x+1)^{4/5}}{4} = \\
 &= \frac{5 \cdot x \cdot (x+1)^{4/5}}{4} - \frac{25 \cdot (x+1)^{9/5}}{36}
 \end{aligned}$$

$$\text{b) } \int \frac{x dx}{\sqrt{x+1}} = \frac{2 \cdot (x+1)^{3/2}}{3} - 2 \cdot \sqrt{x+1} = 2 \cdot x \cdot \sqrt{x+1} - \frac{4 \cdot (x+1)^{3/2}}{3}$$

$$\begin{aligned}
 \text{c) } \int \frac{x dx}{\sqrt[4]{x+1}} &= \frac{4 \cdot (x+1)^{7/4}}{7} - \frac{4 \cdot (x+1)^{3/4}}{3} = \\
 &= \frac{4 \cdot x \cdot (x+1)^{3/4}}{3} - \frac{16 \cdot (x+1)^{7/4}}{21}
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } \int \frac{x dx}{\sqrt[3]{x+1}} &= \frac{3 \cdot (x+1)^{5/3}}{5} - \frac{3 \cdot (x+1)^{2/3}}{2} = \\
 &= \frac{3 \cdot x \cdot (x+1)^{2/3}}{2} - \frac{9 \cdot (x+1)^{5/3}}{10}
 \end{aligned}$$