

Miracle identities
discovered by Jarosław Wróblewski
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Let:

$$\begin{aligned} a_1 &= -28p^2 - 57pq + 16q^2, & a_2 &= -22p^2 - 133pq - 136q^2, & a_3 &= 10p^2 + 57pq + 56q^2, \\ b_1 &= 10p^2 + 7pq + 16q^2, & b_2 &= -28p^2 - 121pq - 136q^2, & b_3 &= -22p^2 - 19pq + 56q^2, \\ c_1 &= -4p^2 + 57pq + 112q^2, & c_2 &= -14p^2 - 57pq - 40q^2, & c_3 &= 34p^2 + 133pq + 88q^2, \\ d_1 &= 34p^2 + 121pq + 112q^2, & d_2 &= -4p^2 - 7pq - 40q^2, & d_3 &= -14p^2 + 19pq + 88q^2. \end{aligned}$$

Then:

$$\begin{aligned} a_1 + a_2 + a_3 &= b_1 + b_2 + b_3, \\ c_1 + c_2 + c_3 &= d_1 + d_2 + d_3, \\ a_1^2 + a_2^2 + a_3^2 &= c_1^2 + c_2^2 + c_3^2, \\ b_1^2 + b_2^2 + b_3^2 &= d_1^2 + d_2^2 + d_3^2, \\ a_1^3 + a_2^3 + a_3^3 &= b_1^3 + b_2^3 + b_3^3, \\ c_1^3 + c_2^3 + c_3^3 &= d_1^3 + d_2^3 + d_3^3, \\ a_1^5 + a_2^5 + a_3^5 + d_1^5 + d_2^5 + d_3^5 &= b_1^5 + b_2^5 + b_3^5 + c_1^5 + c_2^5 + c_3^5, \\ a_1^7 + a_2^7 + a_3^7 + d_1^7 + d_2^7 + d_3^7 &= b_1^7 + b_2^7 + b_3^7 + c_1^7 + c_2^7 + c_3^7. \end{aligned}$$