

**A Collection of  
Numerical Solutions of  
Multigrade Equations  
Related to the  
Prouhet-Tarry-Escott  
Problem**

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# Notes

I am interested only in solutions leading to symmetric PTE solutions, hence I require a multigrade to be satisfied by all exponents of the same parity up to a certain level.

Pure product solution comes from multiplying polynomials of the form

$$x^p - x^{-p}$$

and then extracting solutions in the well known way.

For even powers there is the same number of terms on both sides (I do not discard zero terms).

For odd powers numbers of left and right terms may differ.

Solution code is composed from:

the largest power

total number of terms

the largest term

the second, third ... largest terms of any side (if needed)

## The best results known to me at the moment

Max exponent	Number of terms	Ideal	Known/Ideal
8	10	10	1.000
9	12	11	1.091
10	12	12	1.000
11	20	13	1.538
12	26	14	1.857
13	26	15	1.733
14	30	16	1.875
15	34	17	2.000
16	42	18	2.333
17	48	19	2.526
18	58	20	2.900
19	65	21	3.095
20	70	22	3.182

For solutions of higher degree see **Version 5** and/or the last page of this document and/or the following paper:

Mihai Cipu, *Upper bounds for norms of products of binomials*. LMS Journal of Computation and Mathematics, 7 (2004), pp. 37-49

Solution code: **8.10.313**

**Powers: 2, 4, 6, 8.**

Number of terms: **10**

Number of left terms: **5**

Number of right terms: **5**

**Left terms:**

313, 301, 188, 100, 99

**Right terms:**

308, 307, 180, 131, 71

**Remarks:**

Discovered by Peter Borwein, Petr Lisonek and Colin Percival (2002).

Solution code: **8.10.515**

**Powers: 2, 4, 6, 8.**

Number of terms: **10**

Number of left terms: **5**

Number of right terms: **5**

**Left terms:**

515, 452, 366, 189, 103

**Right terms:**

508, 471, 331, 245, 18

**Remarks:**

Discovered by Peter Borwein, Petr Lisonek and Colin Percival (2002).

Used by Jarosław Wróblewski (November 27, 2009) to produce solution **9.12.1293**.

Solution code: **8.10.23750**

**Powers: 2, 4, 6, 8.**

Number of terms: **10**

Number of left terms: **5**

Number of right terms: **5**

**Left terms:**

23750, 20667, 20449, 11857, 436

**Right terms:**

23738, 20885, 20231, 11881, 12

**Remarks:**

The smallest member of family of solutions discovered by A. Letac in 1940's.

Solution code	Right terms	Left terms
<b>8.12.36</b>	36, 31, 30, 17, 7, 1	35, 34, 27, 19, 4, 3
<b>8.12.62</b>	62, 54, 47, 35, 27, 9	61, 57, 42, 37, 30, 1
<b>8.12.66</b>	66, 59, 57, 55, 28, 16	64, 62, 60, 49, 33, 11
<b>8.12.71</b>	71, 63, 50, 23, 18, 9	69, 67, 42, 37, 6, 5
<b>8.12.74</b>	74, 67, 47, 46, 27, 15	73, 69, 45, 41, 38, 2
<b>8.12.109</b>	109, 97, 91, 88, 54, 30	107, 99, 98, 74, 65, 24
<b>8.12.111</b>	111, 86, 81, 28, 25, 22	110, 94, 63, 57, 4, 1
<b>8.12.113</b>	113, 97, 89, 52, 31, 10	109, 107, 74, 67, 20, 13
<b>8.12.114</b>	114, 97, 89, 70, 43, 24	111, 106, 75, 73, 56, 2
<b>8.12.119.115</b>	119, 97, 94, 92, 45, 30	115, 111, 90, 74, 68, 7
<b>8.12.119.118</b>	119, 94, 85, 43, 38, 37	118, 101, 67, 61, 50, 7
<b>8.12.137</b>	137, 120, 110, 73, 57, 3	135, 127, 97, 88, 45, 18
<b>8.12.139</b>	139, 124, 115, 89, 42, 18	135, 133, 106, 93, 46, 4
<b>8.12.151</b>	151, 139, 117, 58, 55, 28	149, 143, 113, 70, 37, 36
<b>8.12.158</b>	158, 137, 126, 125, 76, 55	154, 148, 127, 106, 95, 45
<b>8.12.163</b>	163, 145, 138, 103, 56, 14	161, 152, 131, 105, 58, 2
<b>8.12.167</b>	167, 150, 122, 89, 71, 47	163, 158, 109, 93, 85, 34
<b>8.12.179.172</b>	179, 151, 150, 108, 67, 38	172, 171, 123, 122, 74, 25
<b>8.12.179.173</b>	179, 149, 139, 65, 63, 57	173, 167, 111, 91, 81, 5
<b>8.12.179.178</b>	179, 142, 132, 83, 59, 35	178, 151, 111, 103, 55, 28
<b>8.12.195</b>	195, 169, 148, 98, 71, 42	190, 182, 127, 111, 84, 13
<b>8.12.211</b>	211, 165, 155, 59, 54, 44	209, 180, 121, 111, 31, 10
<b>8.12.212</b>	212, 189, 167, 114, 83, 10	206, 202, 148, 133, 75, 9
<b>8.12.222</b>	222, 182, 164, 59, 55, 41	220, 193, 146, 97, 26, 21
<b>8.12.229</b>	229, 215, 166, 118, 63, 48	224, 222, 162, 113, 89, 5
<b>8.12.237</b>	237, 206, 195, 88, 49, 41	231, 223, 179, 104, 50, 3
<b>8.12.243</b>	243, 219, 178, 112, 71, 50	240, 226, 167, 126, 67, 43
<b>8.12.245</b>	245, 213, 202, 141, 101, 22	239, 231, 178, 158, 97, 15
<b>8.12.265.256</b>	265, 240, 204, 179, 73, 67	256, 255, 197, 172, 111, 5
<b>8.12.265.257</b>	265, 229, 223, 133, 72, 4	257, 252, 200, 149, 43, 41
<b>8.12.265.264</b>	265, 229, 219, 108, 88, 6	264, 236, 211, 122, 57, 45
<b>8.12.267</b>	267, 245, 177, 104, 79, 4	265, 249, 168, 124, 61, 13
<b>8.12.282.277</b>	282, 237, 223, 122, 107, 53	277, 257, 197, 138, 118, 3
<b>8.12.282.278</b>	282, 229, 215, 107, 54, 47	278, 250, 177, 149, 37, 9
<b>8.12.295</b>	295, 216, 203, 106, 91, 78	294, 232, 169, 126, 125, 13

**Remarks:**

**Powers: 2, 4, 6, 8** with  $6+6=12$  terms.

Results of a selective search by Jarosław Wróblewski (December 2009).

Solution code	Right terms	Left terms
<b>8.12.303</b>	303, 265, 227, 119, 81, 19	291, 289, 191, 167, 45, 7
<b>8.12.325</b>	325, 277, 271, 201, 131, 57	317, 305, 233, 219, 139, 39
<b>8.12.326</b>	326, 273, 243, 227, 112, 89	317, 301, 208, 207, 186, 7
<b>8.12.347</b>	347, 289, 246, 112, 97, 54	343, 306, 194, 192, 43, 41
<b>8.12.358</b>	358, 329, 323, 281, 75, 12	357, 335, 317, 282, 76, 1
<b>8.12.362</b>	362, 295, 291, 163, 118, 81	353, 333, 233, 205, 134, 6
<b>8.12.365</b>	365, 305, 291, 219, 139, 79	355, 339, 241, 229, 181, 15
<b>8.12.370</b>	370, 311, 252, 201, 101, 61	369, 316, 241, 205, 123, 14
<b>8.12.375</b>	375, 325, 266, 184, 162, 61	371, 338, 230, 215, 171, 24
<b>8.12.389</b>	389, 354, 329, 281, 80, 50	379, 375, 304, 295, 94, 14
<b>8.12.392</b>	392, 328, 308, 241, 125, 109	385, 356, 277, 224, 197, 32
<b>8.12.405</b>	405, 362, 351, 205, 162, 49	393, 390, 331, 210, 167, 29
<b>8.12.407.398.383</b>	407, 365, 284, 242, 163, 79	398, 383, 251, 233, 220, 11
<b>8.12.407.398.385</b>	407, 363, 328, 265, 98, 70	398, 385, 293, 287, 120, 22
<b>8.12.407.405</b>	407, 386, 249, 129, 105, 2	405, 389, 243, 154, 74, 27
<b>8.12.412</b>	412, 358, 337, 161, 154, 27	407, 378, 314, 203, 92, 71
<b>8.12.417</b>	417, 347, 277, 184, 169, 64	416, 353, 248, 233, 139, 69
<b>8.12.418</b>	418, 368, 335, 237, 149, 66	402, 401, 302, 253, 165, 16
<b>8.12.421</b>	421, 357, 343, 274, 76, 62	419, 372, 323, 281, 98, 14
<b>8.12.430</b>	430, 351, 344, 203, 179, 82	424, 386, 283, 259, 162, 65
<b>8.12.438</b>	438, 389, 335, 256, 122, 97	430, 409, 302, 277, 151, 48
<b>8.12.449</b>	449, 397, 381, 374, 300, 28	436, 431, 363, 357, 316, 10
<b>8.12.470</b>	470, 396, 382, 257, 183, 61	465, 423, 349, 268, 194, 22
<b>8.12.471</b>	471, 407, 379, 295, 163, 105	453, 449, 335, 303, 209, 35
<b>8.12.509</b>	509, 429, 420, 248, 172, 111	495, 477, 364, 284, 179, 72
<b>8.12.513</b>	513, 410, 336, 218, 109, 77	512, 418, 315, 241, 123, 14
<b>8.12.531</b>	531, 471, 467, 269, 67, 7	523, 501, 441, 277, 41, 37
<b>8.12.538</b>	538, 472, 415, 321, 137, 114	529, 498, 361, 358, 177, 40

**Remarks:**

**Powers: 2, 4, 6, 8** with  $6+6=12$  terms.

Results of a selective search by Jarosław Wróblewski (December 2009).

Solution code: **8.12.541**

**Powers: 2, 4, 6, 8.**

Number of terms: **12**

Number of left terms: **6**

Number of right terms: **6**

**Left terms:**

541, 503, 339, 176, 140, 73

**Right terms:**

532, 517, 305, 251, 96, 31

**Remarks:**

A member of family of solutions discovered by Jarosław Wróblewski (November 2009).

Left side terms:

$$2 a + 5 b + d$$

$$2 a + 5 b - d$$

$$5 a - 2 b + c$$

$$5 a - 2 b - c$$

$$4 a + 6 b$$

$$6 a - 4 b$$

Right side terms:

$$5 a + 2 b + c$$

$$5 a + 2 b - c$$

$$-2 a + 5 b + d$$

$$-2 a + 5 b - d$$

$$6 a + 4 b$$

$$-4 a + 6 b$$

Assume:

$$c^2 = p * a^2 + q * b^2$$

$$d^2 = p * b^2 + q * a^2$$

$$p = -11/5$$

$$q = 64/5$$

This solution is obtained with

$$a = 118$$

$$b = 89$$

$$c = 266$$

$$d = 401$$



Solution code	Right terms	Left terms
<b>8.12.562</b>	562, 487, 466, 439, 233, 225	549, 523, 470, 373, 326, 163
<b>8.12.575</b>	575, 497, 357, 216, 181, 76	573, 504, 323, 281, 140, 71
<b>8.12.583</b>	583, 501, 427, 205, 113, 109	569, 539, 347, 317, 67, 15
<b>8.12.596</b>	596, 446, 445, 243, 242, 111	594, 485, 354, 353, 149, 148
<b>8.12.797</b>	797, 703, 635, 471, 252, 136	771, 760, 567, 508, 289, 37
<b>8.12.890</b>	890, 732, 653, 470, 233, 213	883, 772, 555, 535, 318, 62

**Remarks:**

**Powers: 2, 4, 6, 8** with  $6+6=12$  terms.

Results of a selective search by Jarosław Wróblewski (December 2009).

See the following pages for numerical analysis of **8.12.596**.

## Numerical analysis of **8.12.596**

Performed by Jarosław Wróblewski (December 12, 2009).

Analysis of possible linear constrains in solution **8.12.596** together with the equation for power 2 led me to the following substitutions:

Left side terms:

$$a + 2b + c$$

$$2c + 3x$$

$$c + 3x$$

$$b + c$$

$$b$$

$$a$$

Right side terms:

$$2c + 4x$$

$$2b + c$$

$$a + b + c$$

$$a + b$$

$$c + x$$

$$x$$

with  $(a,b,c,x)=(111,242,1,148)$  in case of solution **8.12.596**.

While after this substitution the multigrade equation **is satisfied for power 2**, equation for **power 4** is quadratic in  $a$  and can be solved as such.

Equations for **powers 6 and 8** happen to have common quadratic factor and will be both satisfied if

$$3b^2 + 3bc - 2c^2 - 8cx - 8x^2 = 0.$$

This can be solved for  $x$ .

Value of  $x$  is rational iff

$$6b(b+c) \tag{8.12.596-1}$$

is a square and value of  $a$  is rational iff

$$2(17b^2 + 17bc - 4c^2) \tag{8.12.596-2}$$

is a square.

With  $b = 2s^2$  and  $c = 3t^2 - b$  the expression (8.12.596-1) is a square, while the expression (8.12.596-2), after dividing by 4, takes form

$$-8s^4 + 75s^2t^2 - 18t^4.$$

In the analized numerical solution we have  $s = 11$  and  $t = 9$ .

For  $(s,t,a,b,c,x) = (9, 17, -738, 162, 705, -582)$  we get solution **8.12.347**.

For  $(s,t,a,b,c,x) = (123, 209, -138504, 30258, 100785, -88953)$  we get  
55358, 46168, 43681, 21763, 10086, 7599  
53767, 51414, 36082, 29651, 3944, 2487

For  $(s,t,a,b,c,x) = (113, 137, -91739, 25538, 30769, -38606)$  we get  
 92886, 81845, 66201, 38606, 35432, 7837  
 91739, 85049, 56307, 54280, 25538, 9894

For  $(s,t,a,b,c,x) = (659, 361, -1409216, 868562, -477599, -118049)$  we get  
 1427394, 1259525, 1018253, 595648, 540654, 118049  
 1409216, 1309345, 868562, 831746, 390963, 149691

For  $(s,t,a,b,c,x) = (907, 417, -2175549, 1645298, -1123631, -5513)$  we get  
 2269314, 2166965, 1653882, 1129144, 530251, 5513  
 2263801, 2175549, 1645298, 1140170, 521667, 8584

For  $(s,t,a,b,c,x) = (849, 1697, -5789991, 1441602, 7197825, -5760042)$  we get  
 3360767, 2879809, 1929997, 1430346, 961492, 480534  
 3360343, 2881506, 1920014, 1449463, 949812, 479261

The above is with the exact notations I have been discovering the family of solutions.  
 A nicer form can be obtained by substitutions:

$$b = A - B$$

$$c = 2B$$

$$X^2 = 3(A^2 - B^2)/8 \quad (8.12.596 - 3)$$

$$Y^2 = (17A^2 - 33B^2)/8 \quad (8.12.596 - 4)$$

$$a = -A + Y$$

$$x = -B + X$$

Now with (8.12.596 - 3) and (8.12.596 - 4) satisfied, we get the following solution for  
**powers 2,4,6,8:**

Left side terms:

$$A + Y$$

$$B + 3 X$$

$$-B + 3 X$$

$$A + B$$

$$A - B$$

$$-A + Y$$

Right side terms:

$$4 X$$

$$2 A$$

$$B + Y$$

$$-B + Y$$

$$B + X$$

$$-B + X$$

Solution code: **8.12.55292**

**Powers: 2, 4, 6, 8.**

Number of terms: **12**

Number of left terms: **6**

Number of right terms: **6**

**Left terms:**

55292, 50841, 50712, 41050, 23681, 11369

**Right terms:**

54280, 54151, 45759, 44489, 20902, 14148

**Remarks:**

A member of family of solutions discovered by Jarosław Wróblewski (November 2009).

Left side terms:

$a + 10 b + d$

$a + 10 b - d$

$10 a - b + c$

$10 a - b - c$

$a + 11 b$

$11 a - b$

Right side terms:

$10 a + b + c$

$10 a + b - c$

$-a + 10 b + d$

$-a + 10 b - d$

$11 a + b$

$-a + 11 b$

Assume:

$$c^2 = p * a^2 + q * b^2$$

$$d^2 = p * b^2 + q * a^2$$

$$p = -27/5$$

$$q = 248/5$$

This solution is obtained with

$a = 9533$

$b = 3439$

$c = 9791$

$d = 66661$

Solution code: **9.12.323**

**Powers: 1, 3, 5, 7, 9.**

Number of terms: **12**

Number of left terms: **6**

Number of right terms: **6**

**Left terms:**

323, 289, 269, 173, 91, 7

**Right terms:**

313, 311, 247, 193, 59, 29

**Remarks:**

Discovered by Chen Shuwen (2000).

Solution code: **9.12.407**

**Powers: 1, 3, 5, 7, 9.**

Number of terms: **12**

Number of left terms: **6**

Number of right terms: **6**

**Left terms:**

407, 347, 341, 181, 163, 23

**Right terms:**

403, 371, 311, 221, 119, 37

**Remarks:**

Discovered by Jarosław Wróblewski (November 28, 2009).

Solution code: **9.12.463**

**Powers: 1, 3, 5, 7, 9.**

Number of terms: **12**

Number of left terms: **6**

Number of right terms: **6**

**Left terms:**

463, 391, 335, 217, 161, 43

**Right terms:**

461, 403, 287, 283, 91, 85

**Remarks:**

Discovered by Jarosław Wróblewski (November 28, 2009).

Solution code: **9.12.1293**

**Powers: 1, 3, 5, 7, 9.**

Number of terms: **12**

Number of left terms: **6**

Number of right terms: **6**

**Left terms:**

1293, 1167, 995, 679, 399, 57

**Right terms:**

1279, 1205, 925, 767, 299, 115

**Remarks:**

Constructed by Jarosław Wróblewski (November 27, 2009) from solution **8.10.515**.



Solution code: **10.12.151**

**Powers: 2, 4, 6, 8, 10.**

Number of terms: **12**

Number of left terms: **6**

Number of right terms: **6**

**Left terms:**

151, 140, 127, 86, 61, 22

**Right terms:**

148, 146, 121, 94, 47, 35

**Remarks:**

Discovered by Nuutti Kuosa (1999) using a computer program written by Jean-Charles Meyrignac, as a single-grade solution to power 10. Four days later Chen Shuwen noticed it was in fact a multigrade.

Solution code: **10.12.1511**

**Powers: 2, 4, 6, 8, 10.**

Number of terms: **12**

Number of left terms: **6**

Number of right terms: **6**

**Left terms:**

1511, 1138, 1075, 700, 622, 107

**Right terms:**

1510, 1180, 953, 886, 413, 293

**Remarks:**

The smallest solution of the infinite family of solutions constructed in:

Ajai Choudhry, Jarosław Wróblewski, *Ideal Solutions of the Tarry-Escott Problem of degree eleven with applications to Sums of Thirteenth Powers*, Hardy-Ramanujan Journal, Vol. 31 (2008) pp. 1-13

The above paper is available at:

<http://www.nias.res.in/hrj/contentsvol31.htm>

Solution code: **10.12.2058**

**Powers: 2, 4, 6, 8, 10.**

Number of terms: **12**

Number of left terms: **6**

Number of right terms: **6**

**Left terms:**

2058, 1896, 1618, 1109, 891, 257

**Right terms:**

2037, 1947, 1514, 1294, 639, 472

**Remarks:**

Discovered by David Broadhurst (2007):

D. Broadhurst, *A Chinese Prouhet-Tarry-Escott solution*,

<http://physics.open.ac.uk/~dbroadhu/cpte.pdf>

The second known solution.

It was later used by Ajai Choudhry and Jarosław Wróblewski to produce an infinite family of solutions:

Ajai Choudhry, Jarosław Wróblewski, *Ideal Solutions of the Tarry-Escott Problem of degree eleven with applications to Sums of Thirteenth Powers*, Hardy-Ramanujan Journal, Vol. 31 (2008) pp. 1-13

The above paper is available at:

<http://www.nias.res.in/hrj/contentsvol31.htm>

Solution code: **10.12.14770**

**Powers: 2, 4, 6, 8, 10.**

Number of terms: **12**

Number of left terms: **6**

Number of right terms: **6**

**Left terms:**

14770, 12638, 11632, 7115, 7043, 929

**Right terms:**

14693, 13165, 10112, 9718, 4054, 3455

**Remarks:**

3rd smallest solution of the infinite family of solutions constructed in:

Ajai Choudhry, Jarosław Wróblewski, *Ideal Solutions of the Tarry-Escott Problem of degree eleven with applications to Sums of Thirteenth Powers*, Hardy-Ramanujan Journal, Vol. 31 (2008) pp. 1-13

The above paper is available at:

<http://www.nias.res.in/hrj/contentsvol31.htm>

Solution code: **10.12.23742**

**Powers: 2, 4, 6, 8, 10.**

Number of terms: **12**

Number of left terms: **6**

Number of right terms: **6**

**Left terms:**

23742, 18687, 18372, 12734, 9611, 349

**Right terms:**

23708, 19653, 16426, 14714, 7713, 3309

**Remarks:**

4th smallest solution of the infinite family of solutions constructed in:

Ajai Choudhry, Jarosław Wróblewski, *Ideal Solutions of the Tarry-Escott Problem of degree eleven with applications to Sums of Thirteenth Powers*, Hardy-Ramanujan Journal, Vol. 31 (2008) pp. 1-13

The above paper is available at:

<http://www.nias.res.in/hrj/contentsvol31.htm>

Solution code: **10.14.68**

**Powers: 2, 4, 6, 8, 10.**

Number of terms: **14**

Number of left terms: **7**

Number of right terms: **7**

**Left terms:**

68, 61, 55, 32, 31, 28, 1

**Right terms:**

67, 64, 49, 44, 23, 20, 17

**Remarks:**

Found by direct search (Jarosław Wróblewski, November 2009) - see solution **10.14.400** for details of a family it belong to.

Solution code: **10.14.400**

**Powers: 2, 4, 6, 8, 10.**

Number of terms: **14**

Number of left terms: **7**

Number of right terms: **7**

**Left terms:**

400, 365, 359, 254, 242, 89, 35

**Right terms:**

395, 383, 341, 271, 230, 70, 64

**Remarks:**

Constructed by Tito Piezas and Jarosław Wróblewski (November 2009):

Left terms:

-15 a - 4 e

15 a - 4 e

8 e

-3 a - 6 y

3 a - 6 y

-12 e + 6 z

12 e + 6 z

Right terms:

-16 e

-12 a + 8 e

12 a + 8 e

-9 a - 6 y

9 a - 6 y

-3 a + 6 z

3 a + 6 z

Constrains:

$$y^2 = a^2 + 52e^2/9$$

$$z^2 = 4a^2 + 4e^2/9$$

The above is as it appeared has appeared in original construction - it can be simplified by dividing corresponding coefficients by a common factor.

Solutions:

(a,e,y,z) = (3,12,29,10) leads to solution **10.14.68**

(a,e,y,z) = (612,105,662,1226) leads to this solution

(a,e,y,z) = (783,168,881,1570) leads to **10.14.4139**

(a,e,y,z) = (989,1980,4861,2378) leads to **10.14.12676**

(a,e,y,z) = (4876,2079,6982,9850) leads to

3502, 3394, 2701, 2355, 1423, 1136, 693

3574, 3131, 3072, 1853, 1745, 1386, 83

$(a,e,y,z) = (13231,10440,28369,27362)$  leads to  
96484, 80075, 69969, 52235, 43507, 27840, 12964  
96431, 80764, 67955, 55680, 41493, 25084, 17045

$(a,e,y,z) = (2941,44460,106909,30218)$  leads to  
238276, 216759, 210877, 118560, 117404, 73985, 44575  
237120, 222641, 204995, 130324, 106796, 63377, 57495



Solution code: **10.14.4139**

**Powers: 2, 4, 6, 8, 10.**

Number of terms: 14

Number of left terms: 7

Number of right terms: 7

**Left terms:**

4139, 3812, 3691, 2545, 2468, 979, 448

**Right terms:**

4111, 3923, 3580, 2684, 2357, 896, 587

**Remarks:**

Constructed by Tito Piezas and Jarosław Wróblewski (November 2009) - see solution 10.14.400 for details.

Solution code: **10.14.12689**

**Powers: 2, 4, 6, 8, 10.**

Number of terms: 14

Number of left terms: 7

Number of right terms: 7

**Left terms:**

12689, 10560, 9236, 6755, 5745, 3767, 1324

**Right terms:**

12676, 10711, 8733, 7585, 5280, 3164, 2305

**Remarks:**

Constructed by Tito Piezas and Jarosław Wróblewski (November 2009) - see solution 10.14.400 for details.

Solution code: **10.16.93**

**Powers: 2, 4, 6, 8, 10.**

Number of terms: **16**

Number of left terms: **8**

Number of right terms: **8**

**Left terms:**

93, 87, 72, 52, 44, 41, 29, 1

**Right terms:**

92, 89, 67, 61, 39, 36, 33, 8

Solution code: **10.16.113**

**Powers: 2, 4, 6, 8, 10.**

Number of terms: **16**

Number of left terms: **8**

Number of right terms: **8**

**Left terms:**

113, 103, 91, 64, 59, 34, 10, 2

**Right terms:**

112, 106, 85, 74, 53, 26, 23, 1

Solution code: **10.16.132**

**Powers: 2, 4, 6, 8, 10.**

Number of terms: **16**

Number of left terms: **8**

Number of right terms: **8**

**Left terms:**

132, 117, 103, 71, 61, 59, 56, 12

**Right terms:**

131, 121, 92, 84, 72, 43, 39, 37

Solution code: **10.16.155**

**Powers: 2, 4, 6, 8, 10.**

Number of terms: **16**

Number of left terms: **8**

Number of right terms: **8**

**Left terms:**

155, 135, 133, 97, 93, 93, 55, 17

**Right terms:**

153, 145, 115, 107, 105, 83, 43, 33

Solution code: **10.16.172**

**Powers: 2, 4, 6, 8, 10.**

Number of terms: **16**

Number of left terms: **8**

Number of right terms: **8**

**Left terms:**

172, 159, 149, 135, 106, 78, 43, 14

**Right terms:**

169, 166, 140, 138, 111, 74, 37, 27

Solution code: **10.16.173**

**Powers: 2, 4, 6, 8, 10.**

Number of terms: **16**

Number of left terms: **8**

Number of right terms: **8**

**Left terms:**

173, 156, 143, 104, 92, 91, 39, 13

**Right terms:**

168, 167, 123, 116, 113, 61, 44, 29



Solution code: **10.16.188**

**Powers: 2, 4, 6, 8, 10.**

Number of terms: **16**

Number of left terms: **8**

Number of right terms: **8**

**Left terms:**

188, 168, 166, 125, 114, 83, 39, 25

**Right terms:**

183, 182, 151, 131, 120, 62, 60, 19

Solution code: **10.16.193**

**Powers: 2, 4, 6, 8, 10.**

Number of terms: **16**

Number of left terms: **8**

Number of right terms: **8**

**Left terms:**

193, 179, 164, 100, 75, 72, 51, 31

**Right terms:**

191, 184, 159, 109, 68, 60, 53, 45

**Remarks:**

By a proper sign changes we can make the solution work for powers 1 and 3:

**Left terms:**

193, -179, 164, -100, 75, 72, 51, -31

**Right terms:**

191, -184, 159, 109, -68, -60, 53, 45

Solution code: **10.16.275**

**Powers: 2, 4, 6, 8, 10.**

Number of terms: **16**

Number of left terms: **8**

Number of right terms: **8**

**Left terms:**

275, 239, 211, 158, 100, 98, 2, 1

**Right terms:**

274, 245, 188, 185, 89, 86, 37, 22

**Remarks:**

Derived from solution **9.12.463**.

Solution code: **10.16.2567**

**Powers: 2, 4, 6, 8, 10.**

Number of terms: **16**

Number of left terms: **8**

Number of right terms: **8**

**Left terms:**

2567, 2339, 2283, 1544, 1426, 710, 479, 237

**Right terms:**

2536, 2449, 2173, 1654, 1347, 631, 510, 347

**Remarks:**

Constructed by Tito Piezas and Jarosław Wróblewski (November 2009).

Solution code: **11.20.107**

**Powers: 1, 3, 5, 7, 9, 11.**

Number of terms: **20**

Number of left terms: **10**

Number of right terms: **10**

**Left terms:**

107, 101, 86, 78, 66, 55, 43, 25, 19, 13

**Right terms:**

106, 103, 81, 79, 73, 50, 38, 30, 27, 6

**Remarks:**

Derived from solution **10.16.93**.

Solution code: **11.20.139**

**Powers: 1, 3, 5, 7, 9, 11.**

Number of terms: **20**

Number of left terms: **11**

Number of right terms: **9**

**Left terms:**

139, 125, 125, 113, 95, 85, 67, 65, 31, 5, 1

**Right terms:**

137, 133, 119, 107, 101, 91, 61, 53, 49

**Remarks:**

Derived from solution **10.14.68**.

Solution code: **11.20.178**

**Powers: 1, 3, 5, 7, 9, 11.**

Number of terms: **20**

Number of left terms: **11**

Number of right terms: **9**

**Left terms:**

178, 167, 154, 119, 94, 88, 67, 49, 20, 8, 5

**Right terms:**

175, 173, 148, 124, 100, 74, 62, 59, 34

**Remarks:**

Derived from solution **10.12.151**.

Solution code: **11.20.199**

**Powers: 1, 3, 5, 7, 9, 11.**

Number of terms: **20**

Number of left terms: **11**

Number of right terms: **9**

**Left terms:**

199, 182, 169, 141, 118, 97, 90, 39, 35, 18, 3

**Right terms:**

194, 193, 149, 147, 139, 78, 70, 66, 55

**Remarks:**

Derived from solution **10.16.173**.



Solution code: **11.20.327**

**Powers: 1, 3, 5, 7, 9, 11.**

Number of terms: **20**

Number of left terms: **10**

Number of right terms: **10**

**Left terms:**

327, 305, 279, 271, 217, 197, 163, 69, 69, 45

**Right terms:**

321, 317, 277, 255, 229, 213, 119, 97, 95, 19

**Remarks:**

Derived from solution **10.12.151**.

Solution code: **11.20.329**

**Powers: 1, 3, 5, 7, 9, 11.**

Number of terms: **20**

Number of left terms: **10**

Number of right terms: **10**

**Left terms:**

329, 307, 281, 265, 199, 161, 149, 71, 67, 43

**Right terms:**

323, 319, 275, 253, 227, 145, 121, 97, 95, 17

**Remarks:**

Derived from solution **10.12.151**.

Solution code: **11.20.431**

**Powers: 1, 3, 5, 7, 9, 11.**

Number of terms: **20**

Number of left terms: **10**

Number of right terms: **10**

**Left terms:**

431, 409, 383, 301, 251, 167, 163, 113, 85, 7

**Right terms:**

425, 421, 371, 317, 223, 199, 151, 125, 43, 35

**Remarks:**

Derived from solution **10.12.151**.

Solution code: **11.20.569**

**Powers: 1, 3, 5, 7, 9, 11.**

Number of terms: **20**

Number of left terms: **10**

Number of right terms: **10**

**Left terms:**

569, 547, 521, 439, 389, 311, 223, 145, 95, 29

**Right terms:**

563, 559, 509, 455, 361, 337, 197, 173, 79, 35

**Remarks:**

Derived from solution **10.12.151**.

Solution code: **11.20.3615**

**Powers: 1, 3, 5, 7, 9, 11.**

Number of terms: **20**

Number of left terms: **10**

Number of right terms: **10**

**Left terms:**

3615, 2869, 2743, 2427, 1993, 1837, 1767, 1313, 379, 233

**Right terms:**

3613, 2953, 2499, 2429, 2365, 1683, 1557, 1419, 651, 7

**Remarks:**

Derived from solution **10.12.1511**.

Solution code: **11.20.5155**

**Powers: 1, 3, 5, 7, 9, 11.**

Number of terms: **20**

Number of left terms: **9**

Number of right terms: **11**

**Left terms:**

5155, 4409, 4283, 3533, 3377, 2347, 1919, 887, 733

**Right terms:**

5153, 4493, 4039, 3905, 2959, 2719, 1547, 1307, 361, 143, 17

**Remarks:**

Derived from solution **10.12.1511**.

Solution code: **11.20.6269**

**Powers: 1, 3, 5, 7, 9, 11.**

Number of terms: **20**

Number of left terms: **10**

Number of right terms: **10**

**Left terms:**

6269, 5945, 5389, 4371, 3935, 2667, 1921, 1741, 435, 371

**Right terms:**

6227, 6047, 5181, 4741, 3431, 3097, 1963, 1209, 1083, 65

**Remarks:**

Derived from solution **10.12.2058**.

Solution code: **11.20.6625**

**Powers: 1, 3, 5, 7, 9, 11.**

Number of terms: **20**

Number of left terms: **11**

Number of right terms: **9**

**Left terms:**

6625, 6301, 5745, 4727, 4291, 3023, 1995, 1385, 519, 291, 79

**Right terms:**

6583, 6403, 5537, 5097, 3787, 3453, 1607, 1283, 1231

**Remarks:**

Derived from solution **10.12.2058**.



Solution code: **11.20.43107**

**Powers: 1, 3, 5, 7, 9, 11.**

Number of terms: **20**

Number of left terms: **9**

Number of right terms: **11**

**Left terms:**

43107, 38843, 36831, 27797, 27653, 15819, 15425, 12763, 5869

**Right terms:**

42953, 39897, 33791, 33003, 21675, 20477, 15973, 9697, 5459, 663, 519

**Remarks:**

Derived from solution **10.12.14770**.

Solution code: **11.20.48287**

**Powers: 1, 3, 5, 7, 9, 11.**

Number of terms: **20**

Number of left terms: **11**

Number of right terms: **9**

**Left terms:**

48287, 44023, 42011, 32977, 32833, 20605, 16889, 7583, 4661, 1477, 689

**Right terms:**

48133, 45077, 38971, 38183, 26855, 25657, 11837, 10793, 6529

**Remarks:**

Derived from solution **10.12.14770**.

Solution code: **11.20.65507**

**Powers: 1, 3, 5, 7, 9, 11.**

Number of terms: **20**

Number of left terms: **9**

Number of right terms: **11**

**Left terms:**

65507, 55397, 54767, 43491, 37245, 29393, 21283, 17325, 14829

**Right terms:**

65439, 57329, 50875, 47451, 33449, 29461, 24641, 19351, 7445, 2597, 1199

**Remarks:**

Derived from solution **10.12.23742**.

Solution code: **11.20.78905**

**Powers: 1, 3, 5, 7, 9, 11.**

Number of terms: **20**

Number of left terms: **10**

Number of right terms: **10**

**Left terms:**

78905, 68795, 68165, 56889, 50643, 32119, 30723, 12199, 7885, 1431

**Right terms:**

78837, 70727, 64273, 60849, 46847, 38039, 24803, 16063, 5323, 1993

**Remarks:**

Derived from solution **10.12.23742**.

Solution code: **11.22.65**

**Powers: 1, 3, 5, 7, 9, 11.**

Number of terms: **22**

Number of left terms: **11**

Number of right terms: **11**

**Left terms:**

65, 60, 59, 47, 45, 40, 30, 18, 16, 10, 4

**Right terms:**

64, 63, 56, 49, 43, 42, 27, 21, 14, 13, 2

**Pure product of:**

1, 2, 3, 4, 5, 7, 9, 11, 13, 16, 17, 19, 23.

Solution code: **12.26.79**

**Powers: 2, 4, 6, 8, 10, 12.**

Number of terms: **26**

Number of left terms: **13**

Number of right terms: **13**

**Left terms:**

79, 76, 72, 69, 58, 53, 48, 44, 26, 21, 17, 14, 11

**Right terms:**

78, 77, 74, 64, 61, 54, 51, 33, 31, 28, 19, 8, 4

**Pure product of:**

1, 2, 5, 6, 7, 8, 9, 11, 13, 15, 17, 19, 20, 25.

**Remarks:**

This solution is taken from the following paper:

Mihai Cipu, *Upper bounds for norms of products of binomials*. LMS Journal of Computation and Mathematics, 7 (2004), pp. 37-49

Solution code: **12.28.81**

**Powers: 2, 4, 6, 8, 10, 12.**

Number of terms: **28**

Number of left terms: **14**

Number of right terms: **14**

**Left terms:**

81, 74, 73, 72, 59, 52, 50, 48, 44, 30, 23, 15, 14, 1

**Right terms:**

80, 78, 71, 69, 62, 54, 49, 47, 40, 33, 25, 18, 4, 4

**Pure product of:**

1, 3, 4, 5, 6, 7, 10, 11, 13, 16, 17, 19, 23, 27.

Solution code: **12.28.82**

**Powers: 2, 4, 6, 8, 10, 12.**

Number of terms: **28**

Number of left terms: **14**

Number of right terms: **14**

**Left terms:**

82, 75, 74, 73, 60, 51, 49, 47, 33, 24, 21, 15, 12, 6

**Right terms:**

81, 79, 72, 70, 63, 54, 45, 44, 36, 27, 18, 17, 9, 5

**Pure product of:**

1, 3, 4, 5, 6, 7, 10, 11, 13, 16, 17, 19, 23, 29.



Solution code: **12.28.119**

**Powers: 2, 4, 6, 8, 10, 12.**

Number of terms: **28**

Number of left terms: **14**

Number of right terms: **14**

**Left terms:**

119, 109, 105, 103, 81, 81, 67, 65, 57, 29, 27, 19, 19, 11

**Right terms:**

117, 115, 101, 99, 89, 79, 73, 51, 49, 45, 41, 7, 3, 1

Solution code: **12.28.169**

**Powers: 2, 4, 6, 8, 10, 12.**

Number of terms: **28**

Number of left terms: **14**

Number of right terms: **14**

**Left terms:**

169, 151, 149, 145, 105, 101, 99, 95, 71, 63, 51, 47, 43, 23

**Right terms:**

167, 161, 139, 135, 133, 89, 85, 83, 81, 79, 61, 39, 33, 25

**Pure product of:**

1, 4, 5, 6, 7, 8, 9, 11, 13, 15, 17, 19, 23, 31.

Solution code: **13.26.173**

**Powers: 1, 3, 5, 7, 9, 11, 13.**

Number of terms: **26**

Number of left terms: **13**

Number of right terms: **13**

**Left terms:**

173, 159, 157, 131, 129, 107, 103, 79, 75, 51, 25, 9, 1

**Right terms:**

171, 167, 141, 139, 137, 97, 93, 89, 85, 43, 19, 15, 3

**Pure product of:**

1, 3, 4, 5, 6, 7, 9, 10, 11, 13, 16, 17, 19, 23, 29.

**Remarks:**

Published by L.J. Lander (1973), *Mathematics of Computation* 27 (122), 1973, p. 397

Solution code: **13.28.191**

**Powers: 1, 3, 5, 7, 9, 11, 13.**

Number of terms: **28**

Number of left terms: **14**

Number of right terms: **14**

**Left terms:**

191, 177, 175, 173, 147, 131, 129, 125, 113, 69, 57, 51, 27, 7

**Right terms:**

189, 185, 171, 167, 153, 137, 123, 119, 115, 71, 67, 37, 21, 17

**Pure product of:**

1, 3, 4, 5, 6, 7, 10, 11, 13, 16, 17, 19, 23, 27, 29.

Solution code: **13.30.69**

**Powers: 1, 3, 5, 7, 9, 11, 13.**

Number of terms: **30**

Number of left terms: **14**

Number of right terms: **16**

**Left terms:**

69, 64, 62, 61, 50, 48, 41, 40, 38, 27, 19, 16, 15, 15

**Right terms:**

68, 67, 60, 59, 54, 46, 45, 34, 32, 31, 31, 10, 9, 8, 6, 5

**Pure product of:**

1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 13, 15, 16, 17, 19.

Solution code: **13.30.129**

**Powers: 1, 3, 5, 7, 9, 11, 13.**

Number of terms: **30**

Number of left terms: **13**

Number of right terms: **17**

**Left terms:**

129, 119, 115, 89, 87, 85, 83, 47, 43, 41, 37, 25, 23

**Right terms:**

127, 125, 103, 101, 95, 81, 63, 59, 51, 49, 19, 17, 9, 9, 7, 5, 3

**Pure product of:**

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 17, 19.

Solution code: **14.30.153**

**Powers: 2, 4, 6, 8, 10, 12, 14.**

Number of terms: **30**

Number of left terms: **15**

Number of right terms: **15**

**Left terms:**

153, 143, 139, 115, 111, 111, 103, 89, 61, 59, 57, 53, 47, 5, 3

**Right terms:**

151, 149, 127, 123, 123, 99, 95, 93, 73, 65, 51, 43, 41, 23, 1

**Pure product of:**

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15, 17, 19, 23.

Solution code: **14.32.187**

**Powers: 2, 4, 6, 8, 10, 12, 14.**

Number of terms: **32**

Number of left terms: **16**

Number of right terms: **16**

**Left terms:**

187, 173, 171, 157, 127, 125, 123, 121, 83, 79, 75, 71, 39, 23, 15, 1

**Right terms:**

185, 181, 159, 155, 151, 115, 111, 107, 103, 99, 61, 57, 37, 33, 17, 13

**Pure product of:**

1, 3, 4, 5, 6, 7, 9, 10, 11, 13, 14, 16, 17, 19, 23, 29.



Solution code: **15.34.189**

**Powers: 1, 3, 5, 7, 9, 11, 13, 15.**

Number of terms: **34**

Number of left terms: **18**

Number of right terms: **16**

**Left terms:**

189, 179, 175, 173, 159, 149, 129, 127, 97, 85, 55, 41, 31, 25, 17, 11, 3, 1

**Right terms:**

187, 185, 171, 169, 161, 155, 121, 119, 117, 69, 63, 39, 37, 21, 19, 13

**Pure product of:**

1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 13, 14, 16, 17, 19, 23, 29.

Solution code: **16.42.95**

**Powers: 2, 4, 6, 8, 10, 12, 14, 16.**

Number of terms: **42**

Number of left terms: **21**

Number of right terms: **21**

**Left terms:**

95, 90, 88, 77, 75, 71, 70, 59, 49, 46, 45, 37, 34, 28, 17, 16, 16, 13, 13, 12, 9

**Right terms:**

94, 93, 82, 81, 79, 68, 67, 56, 55, 53, 35, 32, 31, 29, 27, 26, 20, 7, 6, 5, 0

**Pure product of:**

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 16, 17, 19, 22, 23.

Solution code: **17.48.337**

**Powers: 1, 3, 5, 7, 9, 11, 13, 15, 17.**

Number of terms: **48**

Number of left terms: **24**

Number of right terms: **24**

**Left terms:**

337, 329, 315, 287, 287, 285, 273, 245, 243, 221, 219, 189, 179, 153, 123, 119, 95, 93, 87, 85, 45, 31, 21, 1

**Right terms:**

335, 333, 305, 303, 291, 269, 267, 261, 239, 225, 205, 201, 173, 157, 117, 113, 107, 103, 83, 73, 47, 39, 9, 7

**Pure product of:**

1, 2, 3, 5, 7, 8, 9, 10, 11, 12, 13, 17, 19, 23, 29, 31, 41, 43, 53.

Solution code: **18.58.257**

**Powers: 2, 4, 6, 8, 10, 12, 14, 16, 18.**

Number of terms: **58**

Number of left terms: **29**

Number of right terms: **29**

**Left terms:**

257, 247, 243, 219, 215, 213, 211, 191, 185, 177, 155, 147, 143, 141, 127, 119, 103, 97, 77, 73, 71, 69, 69, 45, 45, 21, 13, 11, 3

**Right terms:**

255, 253, 231, 227, 227, 203, 197, 197, 195, 169, 159, 157, 139, 131, 123, 121, 111, 93, 87, 81, 65, 63, 57, 53, 35, 29, 23, 9, 5

**Pure product of:**

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15, 17, 19, 21, 23, 25, 27, 31.

**Remarks:**

My original own search gave 64 terms as the best result.

This solution is taken from *The Prouhet-Tarry-Escott Problem Revisited* by Peter Borwein and Colin Ignalls (1993).

Solution code: **19.65.143**

**Powers: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19.**

Number of terms: **65**

Number of left terms: **33**

Number of right terms: **32**

**Left terms:**

143, 138, 136, 124, 122, 121, 120, 110, 103, 101, 99, 88, 87, 86, 85, 84, 83, 66, 65, 64, 53, 51, 51, 50, 49, 37, 25, 18, 16, 14, 13, 9, 4

**Right terms:**

142, 141, 130, 128, 128, 116, 114, 113, 109, 95, 94, 93, 93, 91, 81, 80, 76, 75, 61, 59, 58, 57, 56, 45, 43, 34, 24, 22, 20, 19, 10, 8

**Pure product of:**

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31.

**Remarks:**

My original own search gave 74 terms as the best result.

This solution is taken from *The Prouhet-Tarry-Escott Problem Revisited* by Peter Borwein and Colin Ignalls (1993).

Solution code: **20.70.173**

**Powers: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20.**

Number of terms: **70**

Number of left terms: **35**

Number of right terms: **35**

**Left terms:**

173, 168, 166, 163, 156, 154, 153, 134, 127, 125, 115, 106, 96, 95, 86, 86, 84, 74, 72, 70, 67, 60, 56, 46, 45, 36, 27, 26, 25, 24, 17, 15, 13, 7, 3

**Right terms:**

172, 171, 162, 161, 160, 158, 148, 136, 126, 123, 119, 101, 100, 91, 89, 88, 79, 78, 75, 69, 65, 59, 53, 49, 41, 40, 30, 30, 21, 20, 18, 18, 8, 6, 2

**Pure product of:**

1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 37.

**Remarks:**

My original own search gave 88 terms as the best result.

This solution is taken from *The Prouhet-Tarry-Escott Problem Revisited* by Peter Borwein and Colin Ignalls (1993).

## Pure product polynomials of a special kind.

Here are examples of polynomials of the form

$$\prod_{i=1}^k \prod_{p=1}^{a_i} (x^p - x^{-p})$$

with

$$n = a_1 + \dots + a_k$$

and relatively small (the best I could find) half of the sum of absolute values of coefficients (denoted by  $H$ ).

$H$  seems to fit quite well the estimate

$$E(n) = n^{(\log n)/e}$$

with  $\log$  being base  $e$  logarithm.

Although they lead to multigrade equations for exponents up to  $n-2$  with total of  $H$  terms, multigrade solutions obtained that way aren't very interesting, due to a huge number of terms.

$n$	$H$	$a_1, \dots, a_k$	$H/E(n)$
100	5622	91, 8, 1	2.2993
200	58148	185, 13, 2	1.90327
300	221438	275, 21, 3, 1	1.40442
400	713774	370, 25, 4, 1	1.313
500	1685194	465, 28, 6, 1	1.13813
600	3442128	551, 40, 7, 2	0.997715
700	7730080	639, 50, 8, 3	1.07517
800	11745536	733, 55, 8, 3, 1	0.852695
900	20192374	825, 61, 10, 3, 1	0.817166
1000	35087104	919, 65, 12, 3, 1	0.834611
1100	52615906	1013, 71, 12, 3, 1	0.768469
1200	77839804	1106, 79, 10, 4, 1	0.724091
1300	117903318	1195, 85, 15, 4, 1	0.7207
1400	188230374	1291, 89, 15, 4, 1	0.776696
1500	253188340	1374, 104, 16, 4, 2	0.722005
1600	367054114	1442, 125, 24, 6, 2, 1	0.738491
1700	429572776	1538, 132, 21, 6, 2, 1	0.621076
1800	552094724	1634, 136, 21, 6, 2, 1	0.583101
1900	709247742	1738, 132, 21, 6, 2, 1	0.555347
2000	948582974	1823, 147, 21, 6, 2, 1	0.558063
3000	11686822018	2736, 215, 36, 8, 4, 1	0.670286
4000	51936520048	3654, 275, 49, 15, 4, 2, 1	0.530677
5000	198767088094	4555, 357, 63, 16, 6, 2, 1	0.510929
6000	847295849480	5444, 437, 82, 23, 8, 3, 2, 1	0.686346
7000	2212575241804	6358, 508, 93, 26, 9, 3, 2, 1	0.66237
8000	5482592103696	7230, 617, 106, 30, 10, 4, 2, 1	0.683236