

**A Collection of  
Numerical Solutions of  
Multigrade Equations  
Related to the  
Prouhet-Tarry-Escott  
Problem**

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**Upgrade 14 of Version 12, January 4, 2010**

**This Upgrade should be used together with Version 12**

Solution code	Right terms	Left terms
<b>8.12.473</b>	473, 384, 348, 229, 175, 81	471, 400, 319, 252, 177, 59
<b>8.12.544</b>	544, 471, 355, 163, 156, 47	541, 481, 327, 240, 68, 61
<b>8.12.557</b>	557, 502, 458, 308, 233, 57	543, 532, 433, 317, 242, 2
<b>8.12.599</b>	599, 505, 431, 191, 135, 111	591, 535, 369, 295, 89, 1
<b>8.12.623</b>	623, 579, 572, 296, 225, 83	616, 601, 555, 313, 188, 117
<b>8.12.657</b>	657, 600, 530, 433, 219, 109	648, 622, 485, 471, 201, 125
<b>8.12.815</b>	815, 691, 655, 419, 279, 201	785, 771, 569, 409, 401, 45
<b>8.12.828</b>	828, 717, 610, 482, 305, 263	822, 740, 563, 505, 358, 207
<b>8.12.877</b>	877, 843, 699, 625, 199, 127	865, 859, 687, 633, 211, 107
<b>8.12.889</b>	889, 582, 564, 191, 185, 152	888, 628, 471, 361, 70, 31
<b>8.12.915</b>	915, 706, 679, 466, 311, 180	911, 756, 590, 509, 354, 95
<b>8.12.951</b>	951, 763, 695, 690, 463, 116	948, 785, 721, 597, 529, 50
<b>8.12.963</b>	963, 911, 695, 544, 477, 81	951, 929, 673, 563, 480, 51
<b>8.12.1089</b>	1089, 940, 938, 723, 631, 554	1086, 979, 887, 756, 590, 577
<b>8.12.1663</b>	1663, 1523, 1268, 939, 606, 355	1613, 1605, 1186, 993, 652, 269
<b>8.12.1690</b>	1690, 1516, 1229, 795, 511, 406	1670, 1565, 1141, 906, 479, 364
<b>8.12.2107</b>	2107, 1534, 1409, 726, 620, 153	2106, 1563, 1364, 791, 590, 17
<b>8.12.2138</b>	2138, 1847, 1490, 756, 543, 541	2109, 1935, 1228, 1162, 482, 281
<b>8.12.2324</b>	2324, 2137, 1452, 989, 661, 395	2300, 2181, 1244, 1243, 707, 89

**Remarks:**

**Powers: 2, 4, 6, 8** with  $6+6=12$  terms.

Results of a selective search by Jarosław Wróblewski (January 2010).

## FAILED numerical analysis of 8.12.2107

# I NEED HELP !!!

Analysis of possible linear constrains in solution 8.12.2107 led me to the following substitutions:

Left side terms:

$a + b + c + p$   
 $a + b - c + p$   
 $a - b + c + p$   
 $a - b - c + p$   
 $4 p - x$   
 $-2 a + 2 p + 2 x$

Right side terms:

$-a + b + c + p$   
 $-a + b - c + p$   
 $-a - b + c + p$   
 $-a - b - c + p$   
 $4 p + x$   
 $2 a + 2 p - 2 x$

This solution is obtained with

$a=1517/2$   
 $b=573/2$   
 $c=1381/2$   
 $p=743/2$   
 $x=77$

Already works for **power 2**.

$p$  can be obtained from equation for **power 4**. This leaves equations for **powers 6 and 8** in form of homogenous polynomial equations of degree 6 and 9 in  $a, b, c, x$ :

$$\begin{aligned}
 &4a^6 - 4a^4b^2 - 4a^4c^2 + 4a^2b^2c^2 - 64a^5x + 80a^3b^2x - \\
 &32ab^4x + 80a^3c^2x - 32ab^2c^2x - 32ac^4x + 272a^4x^2 - \\
 &208a^2b^2x^2 + 16b^4x^2 - 208a^2c^2x^2 + 96b^2c^2x^2 + \\
 &16c^4x^2 - 484a^3x^3 + 220ab^2x^3 + 220ac^2x^3 + 384a^2x^4 - \\
 &88b^2x^4 - 88c^2x^4 - 133ax^5 + 21x^6 = 0
 \end{aligned}$$

$$\begin{aligned}
&32*a^7*b^2 - 32*a^5*b^4 + 32*a^7*c^2 - 64*a^5*b^2*c^2 + 32*a^3*b^4*c^2 - \\
&32*a^5*c^4 + 32*a^3*b^2*c^4 + 800*a^8*x - 2096*a^6*b^2*x + \\
&1680*a^4*b^4*x - 512*a^2*b^6*x - 2096*a^6*c^2*x + 3520*a^4*b^2*c^2*x - \\
&1232*a^2*b^4*c^2*x + 1680*a^4*c^4*x - 1232*a^2*b^2*c^4*x - 512*a^2*c^6*x - \\
&8080*a^7*x^2 + 13200*a^5*b^2*x^2 - 5696*a^3*b^4*x^2 + 128*a*b^6*x^2 + \\
&13200*a^5*c^2*x^2 - 10192*a^3*b^2*c^2*x^2 + 1280*a*b^4*c^2*x^2 - \\
&5696*a^3*c^4*x^2 + 1280*a*b^2*c^4*x^2 + 128*a*c^6*x^2 + 32420*a^6*x^3 - \\
&32324*a^4*b^2*x^3 + 6320*a^2*b^4*x^3 + 64*b^6*x^3 - 32324*a^4*c^2*x^3 + \\
&14580*a^2*b^2*c^2*x^3 + 960*b^4*c^2*x^3 + 6320*a^2*c^4*x^3 + \\
&960*b^2*c^4*x^3 + 64*c^6*x^3 - 67680*a^5*x^4 + 40896*a^3*b^2*x^4 - \\
&2752*a*b^4*x^4 + 40896*a^3*c^2*x^4 - 6624*a*b^2*c^2*x^4 - 2752*a*c^4*x^4 + \\
&80560*a^4*x^5 - 30416*a^2*b^2*x^5 + 80*b^4*x^5 - 30416*a^2*c^2*x^5 + \\
&480*b^2*c^2*x^5 + 80*c^4*x^5 - 56740*a^3*x^6 + 13192*a*b^2*x^6 + \\
&13192*a*c^2*x^6 + 23665*a^2*x^7 - 2484*b^2*x^7 - 2484*c^2*x^7 - \\
&5560*a*x^8 + 615*x^9 = 0
\end{aligned}$$

**I do not know what to do next !!!**