

**A Collection of
Numerical Solutions of
Multigrade Equations
Related to the
Prouhet-Tarry-Escott
Problem**

by Jarosław Wróblewski

jwr@math.uni.wroc.pl

<http://www.math.uni.wroc.pl/~jwr/PTE>

Upgrade 15 of Version 12, January 8, 2010

This Upgrade should be used together with Version 12

Solution code	Left terms Right terms	Remarks
8.12.36	36, 31, 30, 17, 7, 1 35, 34, 27, 19, 4, 3	Family of 8.12.797
8.12.62	62, 54, 47, 35, 27, 9 61, 57, 42, 37, 30, 1	
8.12.66	66, 59, 57, 55, 28, 16 64, 62, 60, 49, 33, 11	
8.12.71	71, 63, 50, 23, 18, 9 69, 67, 42, 37, 6, 5	
8.12.74	74, 67, 47, 46, 27, 15 73, 69, 45, 41, 38, 2	
8.12.109	109, 97, 91, 88, 54, 30 107, 99, 98, 74, 65, 24	
8.12.111	111, 86, 81, 28, 25, 22 110, 94, 63, 57, 4, 1	
8.12.113	113, 97, 89, 52, 31, 10 109, 107, 74, 67, 20, 13	
8.12.114	114, 97, 89, 70, 43, 24 111, 106, 75, 73, 56, 2	
8.12.119.115	119, 97, 94, 92, 45, 30 115, 111, 90, 74, 68, 7	
8.12.119.118	119, 94, 85, 43, 38, 37 118, 101, 67, 61, 50, 7	
8.12.137	137, 120, 110, 73, 57, 3 135, 127, 97, 88, 45, 18	
8.12.139	139, 124, 115, 89, 42, 18 135, 133, 106, 93, 46, 4	Family of 8.12.797
8.12.151	151, 139, 117, 58, 55, 28 149, 143, 113, 70, 37, 36	
8.12.158	158, 137, 126, 125, 76, 55 154, 148, 127, 106, 95, 45	
8.12.163	163, 145, 138, 103, 56, 14 161, 152, 131, 105, 58, 2	Family of 8.12.797
8.12.167	167, 150, 122, 89, 71, 47 163, 158, 109, 93, 85, 34	
8.12.179.172	179, 151, 150, 108, 67, 38 172, 171, 123, 122, 74, 25	
8.12.179.173	179, 149, 139, 65, 63, 57 173, 167, 111, 91, 81, 5	
8.12.179.178	179, 142, 132, 83, 59, 35 178, 151, 111, 103, 55, 28	

Solution code	Left terms Right terms	Remarks
8.12.195	195, 169, 148, 98, 71, 42 190, 182, 127, 111, 84, 13	Family of 8.12.797
8.12.211	211, 165, 155, 59, 54, 44 209, 180, 121, 111, 31, 10	Family of 8.12.797
8.12.212	212, 189, 167, 114, 83, 10 206, 202, 148, 133, 75, 9	
8.12.222	222, 182, 164, 59, 55, 41 220, 193, 146, 97, 26, 21	
8.12.229	229, 215, 166, 118, 63, 48 224, 222, 162, 113, 89, 5	
8.12.237	237, 206, 195, 88, 49, 41 231, 223, 179, 104, 50, 3	
8.12.243	243, 219, 178, 112, 71, 50 240, 226, 167, 126, 67, 43	
8.12.245	245, 213, 202, 141, 101, 22 239, 231, 178, 158, 97, 15	
8.12.265.256	265, 240, 204, 179, 73, 67 256, 255, 197, 172, 111, 5	
8.12.265.257	265, 229, 223, 133, 72, 4 257, 252, 200, 149, 43, 41	
8.12.265.264	265, 229, 219, 108, 88, 6 264, 236, 211, 122, 57, 45	
8.12.267	267, 245, 177, 104, 79, 4 265, 249, 168, 124, 61, 13	
8.12.282.277	282, 237, 223, 122, 107, 53 277, 257, 197, 138, 118, 3	
8.12.282.278	282, 229, 215, 107, 54, 47 278, 250, 177, 149, 37, 9	
8.12.295	295, 216, 203, 106, 91, 78 294, 232, 169, 126, 125, 13	Family of 8.12.797
8.12.303	303, 265, 227, 119, 81, 19 291, 289, 191, 167, 45, 7	
8.12.325	325, 277, 271, 201, 131, 57 317, 305, 233, 219, 139, 39	
8.12.326	326, 273, 243, 227, 112, 89 317, 301, 208, 207, 186, 7	
8.12.347	347, 289, 246, 112, 97, 54 343, 306, 194, 192, 43, 41	Family of 8.12.797
8.12.358	358, 329, 323, 281, 75, 12 357, 335, 317, 282, 76, 1	Family of 8.12.797

Solution code	Left terms Right terms	Remarks
8.12.362	362, 295, 291, 163, 118, 81 353, 333, 233, 205, 134, 6	Family of 8.12.890
8.12.365	365, 305, 291, 219, 139, 79 355, 339, 241, 229, 181, 15	
8.12.370	370, 311, 252, 201, 101, 61 369, 316, 241, 205, 123, 14	
8.12.375	375, 325, 266, 184, 162, 61 371, 338, 230, 215, 171, 24	
8.12.389	389, 354, 329, 281, 80, 50 379, 375, 304, 295, 94, 14	Family of 8.12.797
8.12.392	392, 328, 308, 241, 125, 109 385, 356, 277, 224, 197, 32	
8.12.405	405, 362, 351, 205, 162, 49 393, 390, 331, 210, 167, 29	
8.12.407.398.383	407, 365, 284, 242, 163, 79 398, 383, 251, 233, 220, 11	
8.12.407.398.385	407, 363, 328, 265, 98, 70 398, 385, 293, 287, 120, 22	Family of 8.12.797
8.12.407.405	407, 386, 249, 129, 105, 2 405, 389, 243, 154, 74, 27	
8.12.412	412, 358, 337, 161, 154, 27 407, 378, 314, 203, 92, 71	
8.12.417	417, 347, 277, 184, 169, 64 416, 353, 248, 233, 139, 69	Family of 8.12.797
8.12.418	418, 368, 335, 237, 149, 66 402, 401, 302, 253, 165, 16	Family of 8.12.797
8.12.421	421, 357, 343, 274, 76, 62 419, 372, 323, 281, 98, 14	Family of 8.12.797
8.12.430	430, 351, 344, 203, 179, 82 424, 386, 283, 259, 162, 65	
8.12.438	438, 389, 335, 256, 122, 97 430, 409, 302, 277, 151, 48	
8.12.449	449, 397, 381, 374, 300, 28 436, 431, 363, 357, 316, 10	
8.12.470	470, 396, 382, 257, 183, 61 465, 423, 349, 268, 194, 22	Family of 8.12.890
8.12.471	471, 407, 379, 295, 163, 105 453, 449, 335, 303, 209, 35	
8.12.473	473, 384, 348, 229, 175, 81 471, 400, 319, 252, 177, 59	January 2010

Solution code	Left terms Right terms	Remarks
8.12.509	509, 429, 420, 248, 172, 111 495, 477, 364, 284, 179, 72	
8.12.513	513, 410, 336, 218, 109, 77 512, 418, 315, 241, 123, 14	
8.12.531	531, 471, 467, 269, 67, 7 523, 501, 441, 277, 41, 37	
8.12.538	538, 472, 415, 321, 137, 114 529, 498, 361, 358, 177, 40	Family of 8.12.797
8.12.541	541, 503, 339, 176, 140, 73 532, 517, 305, 251, 96, 31	Family of 8.12.541
8.12.544	544, 471, 355, 163, 156, 47 541, 481, 327, 240, 68, 61	January 2010
8.12.557	557, 502, 458, 308, 233, 57 543, 532, 433, 317, 242, 2	January 2010
8.12.562	562, 487, 466, 439, 233, 225 549, 523, 470, 373, 326, 163	
8.12.575.565	575, 529, 493, 436, 93, 72 565, 552, 459, 457, 116, 23	Family of 8.12.797
8.12.575.573	575, 497, 357, 216, 181, 76 573, 504, 323, 281, 140, 71	Family of 8.12.797
8.12.583	583, 501, 427, 205, 113, 109 569, 539, 347, 317, 67, 15	
8.12.596	596, 446, 445, 243, 242, 111 594, 485, 354, 353, 149, 148	Family of 8.12.797
8.12.599	599, 505, 431, 191, 135, 111 591, 535, 369, 295, 89, 1	January 2010
8.12.617	617, 513, 460, 347, 155, 102 615, 527, 437, 358, 180, 53	Family of 8.12.797
8.12.623	623, 579, 572, 296, 225, 83 616, 601, 555, 313, 188, 117	January 2010
8.12.657	657, 600, 530, 433, 219, 109 648, 622, 485, 471, 201, 125	January 2010
8.12.751	751, 589, 507, 322, 242, 160 749, 608, 465, 347, 286, 82	Family of 8.12.797
8.12.797	797, 703, 635, 471, 252, 136 771, 760, 567, 508, 289, 37	Family of 8.12.797
8.12.815	815, 691, 655, 419, 279, 201 785, 771, 569, 409, 401, 45	January 2010
8.12.828	828, 717, 610, 482, 305, 263 822, 740, 563, 505, 358, 207	January 2010

Solution code	Left terms Right terms	Remarks
8.12.877	877, 843, 699, 625, 199, 127 865, 859, 687, 633, 211, 107	January 2010
8.12.889	889, 582, 564, 191, 185, 152 888, 628, 471, 361, 70, 31	January 2010
8.12.890	890, 732, 653, 470, 233, 213 883, 772, 555, 535, 318, 62	Family of 8.12.890
8.12.911	911, 785, 744, 409, 121, 80 880, 865, 664, 471, 41, 31	Family of 8.12.797
8.12.915	915, 706, 679, 466, 311, 180 911, 756, 590, 509, 354, 95	January 2010
8.12.951	951, 763, 695, 690, 463, 116 948, 785, 721, 597, 529, 50	January 2010
8.12.963	963, 911, 695, 544, 477, 81 951, 929, 673, 563, 480, 51	January 2010
8.12.992	992, 872, 769, 445, 281, 51 961, 941, 680, 547, 192, 89	Family of 8.12.797
8.12.1024	1024, 935, 759, 461, 230, 54 990, 989, 705, 529, 176, 34	Family of 8.12.797
8.12.1089	1089, 940, 938, 723, 631, 554 1086, 979, 887, 756, 590, 577	January 2010
8.12.1097	1097, 895, 697, 426, 396, 313 1093, 918, 605, 499, 492, 179	Family of 8.12.797
8.12.1355	1355, 1207, 1074, 892, 266, 259 1340, 1253, 994, 941, 361, 102	Family of 8.12.797
8.12.1356	1356, 1189, 1054, 785, 401, 270 1324, 1271, 919, 870, 486, 85	Family of 8.12.797
8.12.1427	1427, 1107, 961, 590, 462, 316 1423, 1148, 869, 645, 558, 146	Family of 8.12.797
8.12.1507	1507, 1204, 1083, 431, 410, 289 1493, 1292, 861, 794, 215, 121	Family of 8.12.797
8.12.1663	1663, 1523, 1268, 939, 606, 355 1613, 1605, 1186, 993, 652, 269	January 2010
8.12.1690	1690, 1516, 1229, 795, 511, 406 1670, 1565, 1141, 906, 479, 364	January 2010
8.12.1691	1691, 1431, 1417, 766, 182, 168 1638, 1599, 1249, 872, 53, 14	Family of 8.12.797
8.12.1877	1877, 1587, 1462, 745, 361, 236 1823, 1748, 1226, 1003, 129, 125	Family of 8.12.797
8.12.2107	2107, 1534, 1409, 726, 620, 153 2106, 1563, 1364, 791, 590, 17	January 2010

Solution code	Left terms Right terms	Remarks
8.12.2132	2132, 1850, 1803, 1037, 291, 29 2103, 1972, 1681, 1095, 169, 122	Family of 8.12.797
8.12.2138	2138, 1847, 1490, 756, 543, 541 2109, 1935, 1228, 1162, 482, 281	January 2010
8.12.2160	2160, 1927, 1751, 1401, 553, 352 2103, 2057, 1575, 1504, 656, 103	Family of 8.12.797
8.12.2207	2207, 1625, 1495, 708, 578, 343 2203, 1700, 1357, 917, 507, 130	Family of 8.12.797
8.12.2215	2215, 1938, 1891, 1172, 1031, 94 2209, 1985, 1844, 1178, 1037, 6	Family of 8.12.797
8.12.2324	2324, 2137, 1452, 989, 661, 395 2300, 2181, 1244, 1243, 707, 89	January 2010
8.12.2411	2411, 2147, 2030, 1528, 807, 234 2373, 2264, 1913, 1566, 845, 38	Family of 8.12.797
8.12.2548	2548, 2257, 2166, 1565, 941, 182 2527, 2348, 2075, 1586, 962, 21	Family of 8.12.797
8.12.2698	2698, 2244, 2241, 1163, 341, 338 2582, 2574, 1903, 1411, 124, 3	Family of 8.12.797
8.12.2729	2729, 2363, 2021, 1558, 684, 615 2705, 2451, 1836, 1679, 893, 278	Family of 8.12.797
8.12.3086	3086, 2693, 2313, 1236, 835, 734 2997, 2900, 2003, 1614, 926, 151	January 2010
8.12.3169	3169, 2877, 2653, 2264, 631, 448 3101, 3032, 2429, 2401, 768, 137	Family of 8.12.797
8.12.3391	3391, 2924, 2883, 1631, 243, 202 3348, 3126, 2681, 1717, 43, 41	Family of 8.12.797
8.12.3745	3745, 3233, 2813, 1772, 1443, 840 3653, 3480, 2393, 2037, 1708, 265	Family of 8.12.797
8.12.3871	3871, 2635, 2464, 1403, 1047, 648 3867, 2783, 2135, 1736, 1088, 171	Family of 8.12.797
8.12.4488	4488, 4004, 3491, 2075, 1137, 169 4319, 4316, 3179, 2413, 825, 312	Family of 8.12.797
8.12.4535	4535, 3850, 2604, 1741, 1474, 679 4529, 3875, 2401, 2134, 1246, 660	Family of 8.12.797
8.12.4679	4679, 4035, 3503, 2188, 1813, 1064 4567, 4340, 2971, 2527, 2152, 339	Family of 8.12.797
8.12.4900	4900, 4471, 4107, 3589, 793, 728 4835, 4641, 3848, 3743, 1052, 259	Family of 8.12.797
8.12.5212	5212, 4477, 3857, 2253, 2145, 1240 5097, 4805, 3237, 2660, 2552, 407	Family of 8.12.797

Solution code	Left terms Right terms	Remarks
8.12.5357	5357, 3703, 3610, 1734, 1641, 1424 5344, 4046, 2735, 2622, 1969, 93	Family of 8.12.797
8.12.5419	5419, 4440, 3721, 2674, 1689, 970 5410, 4504, 3589, 2751, 1830, 719	Family of 8.12.797
8.12.5443	5443, 4879, 4476, 3623, 1378, 806 5282, 5222, 4073, 3844, 1599, 221	Family of 8.12.797
8.12.5666	5666, 4732, 4218, 3209, 1431, 917 5649, 4847, 4028, 3301, 1638, 514	Family of 8.12.797
8.12.5693	5693, 3887, 3588, 2104, 1805, 449 5692, 3945, 3496, 2197, 1783, 299	Family of 8.12.797
8.12.5711	5711, 5125, 4544, 2689, 840, 259 5600, 5384, 4285, 2911, 581, 111	Family of 8.12.797
8.12.5922	5922, 5183, 4688, 3043, 2395, 990 5680, 5678, 4193, 3285, 2637, 242	Family of 8.12.797
8.12.6211	6211, 5746, 5391, 4820, 949, 710 6101, 5990, 5041, 5036, 1170, 221	Family of 8.12.797
8.12.6980	6980, 6109, 5329, 4161, 1651, 1560 6889, 6396, 4745, 4549, 2235, 584	Family of 8.12.797
8.12.7192	7192, 6332, 4685, 3329, 2799, 797 7173, 6395, 4393, 3844, 2488, 841	Family of 8.12.797
8.12.7447	7447, 6588, 6413, 4670, 2723, 350 7420, 6763, 6238, 4697, 2750, 27	Family of 8.12.797
8.12.7772	7772, 6606, 5591, 3219, 2993, 893 7708, 6879, 4779, 4489, 2117, 1102	Family of 8.12.797
8.12.7826	7826, 7521, 7391, 7039, 695, 260 7780, 7651, 7261, 7085, 741, 46	Family of 8.12.797
8.12.8648	8648, 7276, 6947, 3751, 2585, 573 8404, 8075, 5819, 4897, 1457, 1128	Family of 8.12.797
8.12.8943	8943, 7577, 5821, 4270, 3364, 1309 8932, 7634, 5579, 4673, 3111, 1355	Family of 8.12.797
8.12.9568	9568, 8869, 6731, 4395, 3152, 389 9376, 9179, 6224, 5173, 2645, 507	Family of 8.12.797
8.12.9947	9947, 8251, 6627, 3901, 3320, 3248 9875, 8584, 5264, 5003, 4683, 1363	Family of 8.12.797
8.12.10247	10247, 8427, 8414, 4222, 1469, 1456 9883, 9646, 6958, 5424, 601, 13	Family of 8.12.797
8.12.10569	10569, 8729, 6929, 4639, 3600, 2680 10529, 8944, 6264, 5129, 4305, 1625	Family of 8.12.797
8.12.11711	11711, 9272, 7740, 3933, 2653, 2401 11673, 9576, 6923, 5339, 2135, 1532	Family of 8.12.797

Solution code	Left terms Right terms	Remarks
8.12.11887	11887, 8384, 7921, 3936, 3473, 3343 11857, 9039, 6191, 5696, 4448, 463	Family of 8.12.797
8.12.12005	12005, 9436, 8519, 5249, 4590, 2166 11970, 9839, 7415, 6756, 3451, 2534	Family of 8.12.797
8.12.14076	14076, 12190, 12189, 7037, 137, 1 14075, 12258, 12121, 7039, 69, 68	Family of 8.12.797
8.12.55292	55292, 50841, 50712, 41050, 23681, 11369 54280, 54151, 45759, 44489, 20902, 14148	Family of 8.12.55292
8.12.55358	55358, 46168, 43681, 21763, 10086, 7599 53767, 51414, 36082, 29651, 3944, 2487	Family of 8.12.596
8.12.92886	92886, 81845, 66201, 38606, 35432, 7837 91739, 85049, 56307, 54280, 25538, 9894	Family of 8.12.596
8.12.1427394	1427394, 1259525, 1018253, 595648, 540654, 118049 1409216, 1309345, 868562, 831746, 390963, 149691	Family of 8.12.596
8.12.2269314	2269314, 2166965, 1653882, 1129144, 530251, 5513 2263801, 2175549, 1645298, 1140170, 521667, 8584	Family of 8.12.596
8.12.3360767	3360767, 2879809, 1929997, 1430346, 961492, 480534 3360343, 2881506, 1920014, 1449463, 949812, 479261	Family of 8.12.596

Remarks:

Powers: 2, 4, 6, 8 with $6+6=12$ terms.

Family **8.12.797** has a lot of subfamilies, one of them is **8.12.596**.

FAILED numerical analysis of 8.12.2107

I NEED HELP !!!

Analysis of possible linear constrains in solution 8.12.2107 led me to the following substitutions:

Left side terms:

$a + b + c + p$
 $a + b - c + p$
 $a - b + c + p$
 $a - b - c + p$
 $4 p - x$
 $-2 a + 2 p + 2 x$

Right side terms:

$-a + b + c + p$
 $-a + b - c + p$
 $-a - b + c + p$
 $-a - b - c + p$
 $4 p + x$
 $2 a + 2 p - 2 x$

This solution is obtained with

$a=1517/2$
 $b=573/2$
 $c=1381/2$
 $p=743/2$
 $x=77$

Already works for **power 2**.

p can be obtained from equation for **power 4**. This leaves equations for **powers 6 and 8** in form of homogenous polynomial equations of degree 6 and 9 in a, b, c, x :

$$\begin{aligned}
 &4a^6 - 4a^4b^2 - 4a^4c^2 + 4a^2b^2c^2 - 64a^5x + 80a^3b^2x - \\
 &32ab^4x + 80a^3c^2x - 32ab^2c^2x - 32ac^4x + 272a^4x^2 - \\
 &208a^2b^2x^2 + 16b^4x^2 - 208a^2c^2x^2 + 96b^2c^2x^2 + \\
 &16c^4x^2 - 484a^3x^3 + 220ab^2x^3 + 220ac^2x^3 + 384a^2x^4 - \\
 &88b^2x^4 - 88c^2x^4 - 133ax^5 + 21x^6 = 0
 \end{aligned}$$

$$\begin{aligned}
& 32a^7b^2 - 32a^5b^4 + 32a^7c^2 - 64a^5b^2c^2 + 32a^3b^4c^2 - \\
& 32a^5c^4 + 32a^3b^2c^4 + 800a^8x - 2096a^6b^2x + \\
& 1680a^4b^4x - 512a^2b^6x - 2096a^6c^2x + 3520a^4b^2c^2x - \\
& 1232a^2b^4c^2x + 1680a^4c^4x - 1232a^2b^2c^4x - 512a^2c^6x - \\
& 8080a^7x^2 + 13200a^5b^2x^2 - 5696a^3b^4x^2 + 128ab^6x^2 + \\
& 13200a^5c^2x^2 - 10192a^3b^2c^2x^2 + 1280ab^4c^2x^2 - \\
& 5696a^3c^4x^2 + 1280ab^2c^4x^2 + 128ac^6x^2 + 32420a^6x^3 - \\
& 32324a^4b^2x^3 + 6320a^2b^4x^3 + 64b^6x^3 - 32324a^4c^2x^3 + \\
& 14580a^2b^2c^2x^3 + 960b^4c^2x^3 + 6320a^2c^4x^3 + \\
& 960b^2c^4x^3 + 64c^6x^3 - 67680a^5x^4 + 40896a^3b^2x^4 - \\
& 2752ab^4x^4 + 40896a^3c^2x^4 - 6624ab^2c^2x^4 - 2752ac^4x^4 + \\
& 80560a^4x^5 - 30416a^2b^2x^5 + 80b^4x^5 - 30416a^2c^2x^5 + \\
& 480b^2c^2x^5 + 80c^4x^5 - 56740a^3x^6 + 13192ab^2x^6 + \\
& 13192ac^2x^6 + 23665a^2x^7 - 2484b^2x^7 - 2484c^2x^7 - \\
& 5560ax^8 + 615x^9 = 0
\end{aligned}$$

I do not know what to do next !!!

Tito Piezas simplified the system by substitutions:

$$\begin{aligned}
\{a, x\} &= \{u+v, -u+v\} \\
\{b, c\} &= \{r+s, r-s\}
\end{aligned}$$

then $\{p, r, s\}$ can be expressed in two free parameters $\{u, v\}$ as

$$\begin{aligned}
8p^2 &= 5u^2+3v^2 \\
32r^2 &= 65u^2-40uv+15v^2+4w \\
32s^2 &= 65u^2-40uv+15v^2-4w
\end{aligned}$$

where

$$w^2 = 153u^4 - 108u^3v + 38u^2v^2 - 28uv^3 + 9v^4$$

but this indicates that we do not arrive at an elliptic curve.

I have noticed that this solution leads to a solution for powers 1,3,5,7 with 10 terms:
(-3469, -2383, -1983, -897, -437, 589, 709, 2075, 2325, 3471)