# Seminarium geometrów

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## Approximation of $C^1$ mappings with non-maximal rank of the derivative.

Abstract: Let $\Omega \subset \mathbb{R}^n$ be open, $n \geqslant 2$. In 2016, Jacek Gałęski conjectured that Any function $f \in C^1(\Omega, \mathbb{R}^n)$ satisfying $\text{rank} Df \leqslant m < n$ in $\Omega$ can be uniformly approximated by smooth $f_k$ satisfying the same assumption on the rank of the derivative: $\text{rank} Df_k \leqslant m$ in $\Omega$.

Two years later, together with Piotr Hajłasz, we disproved this conjecture, constructing (an infinite series of) examples of $C^1$ mappings with non-maximal rank of the derivative, that cannot be approximated – even locally – by smooth maps with the same restriction on the rank of the derivative. The construction is based on differential topology tools and techniques. However, the simplest of our examples is $f \in C^1(\mathbb{R}^5, \mathbb{R}^5)$ with $\text{rank} Df \leqslant 3$, and the conjecture remains open for an infinite number of pairs $(n, m)$.

Recently, we gave a positive answer for $m = 1$: Any $f \in C^1(\Omega, \mathbb{R}^n)$ with $\text{rank} Df \leqslant 1$ in $\Omega$ can be uniformly approximated by smooth maps $f_k$ with $\text{rank} Df_k \leqslant 1$ in $\Omega$. This time, the proof employs techniques of analysis on metric spaces, in particular the theory of $\mathbb{R}$-trees.

*streaming via ZOOM:*

Meeting ID: 967 6507 7409

Meeting password: "GS"(two letters) followed by the Euler characteristic of the closed orientable surface of genus 89.