We work in a monster model $\mathcal{C}$ of a complete theory $T$.

**Problem 1.** Let $M_n$, $n \in \omega$, be finite $L$-structures, and $\mu_n$ be the normalized counting measure on $M_n$. Let $M := \prod_n M_n/D$ be an ultraproduct. For $b \in M$ take any representative $(b_0, b_1, \ldots) \in \prod_n M_n$. Define $\mu(\varphi(x, b)) := \lim_D \mu_n(\varphi(x, b_n))$.

(i) Prove that this is a well-defined Keisler measure over $M$.
(ii) Let $L^+$ be the original language $L$ expanded by the new sort $S$ carrying $+$ and $\leq$, as well as function symbols $f_\varphi(y)$ from the original sort to $S$ for all $\varphi(x, y) \in L$ (where $|x| = 1$). Each $M_n$ can be naturally expanded to an $L^+$-structure $M_n^+$, by interpreting $S$ as $[0, 1]$, $+$ as addition modulo 1, $\leq$ as the usual order, and $f_\varphi(b) := \mu_n(\varphi(M_n, b))$. Define $M^+ := \prod M_n^+/D$ (so $M^+$ has sorts $M$ and a nonstandard interval $[0, 1]^*$). Check that $\mu(\varphi(x, b)) = \text{st}(f_\varphi(b))$.

**Problem 2.** Prove Claim 1 from page 55 of the notes.

**Problem 3.** Prove Claim 2 from page 55 of the notes.

**Problem 4.** Show that the function $\mu'_x$ obtained in Construction $(\ast)$ on page 56 is a Keisler measure extending $\mu_x$.

**Problem 5.** Let $\mu$ be a Keisler measure over $A$. Let $U \subseteq S_x(A)$ be open. Show that $\inf\{\mu_x([\psi(x)] \cap U) : -\psi(x) \subseteq U\} = 0$.

**Problem 6.** Assume $T$ has NIP. Let $\mu \in M_x(M)$. Prove that there does not exist $\varphi(x, y) \in L$, $(b_i)_{i \in \omega} \subseteq M$, and $\epsilon > 0$ such that for all distinct $i, j < \omega$, $\mu(\varphi(x, b_i) \land \varphi(x, b_j)) \geq \epsilon$.

*Hint.* Use Construction $(\ast)$, Lemma 1 from p. 58, and NIP.

**Problem 7.** Assume $T$ has NIP. Let $\mu \in M_x(\mathcal{C})$.

(i) Prove that $L_x(\mathcal{C})/\sim_\mu$ is of bounded size, and find a bound in terms of $|T|$.
(ii) Prove that $S(\mu)$ is of bounded size, and find a bound in terms of $|T|$.