We work in a monster model $\mathfrak{C}$ of a complete theory $T$; $G = G(\mathfrak{C})$ is a $\emptyset$-definable group.

**Problem 1.** Let $M \prec N \prec \mathfrak{C}$, where $N$ is $|M|^+$-saturated. Assume that $p \in S_G(M)$ satisfies that $g p$ does not fork over $M$ for all $g \in G(N)$. Prove that $p$ extends to a global type which is (strongly) $f$-generic over $M$.

**Problem 2.** Suppose $G$ is definably amenable in all reducts of $\mathfrak{C}$ to countable sublanguages in which $G$ remains to be $\emptyset$-definable. Prove that $G$ is definably amenable.

**Problem 3.** Let $H < G$ be $\emptyset$-definable.
(i) Prove that if $G$ is definably amenable, then so is $G/H$.
(ii) Assume NIP. Prove that if $H$ and $G/H$ are both definably amenable, then so is $G$.

**Problem 4.** Assume NIP. Prove that a definably amenable group $G$ admits a global Keisler measure which is both left and right invariant.

**Hint.** First, observe that there is a left invariant and invariant under $\text{Aut}(\mathfrak{C}/M)$ Keisler measure $\mu$ on $G$. Define $\mu^{-1}(X) := \mu(X^{-1})$. Check that $\mu^{-1}$ is a right invariant and invariant under $\text{Aut}(\mathfrak{C}/M)$ Keisler measure on $G$. Put $\nu(\varphi(x)) := \mu \otimes \mu^{-1}(\varphi(x \cdot y))$ and show that it is as required.

**Problem 5.** Let $M := (\mathbb{R}, +, \cdot)$ and $G(M) := (\mathbb{R}, +) \times \{-1, 1\}$ with $(x_0, \epsilon_0)(x_1, \epsilon_1) := (x_0 + \epsilon_0 x_1, \epsilon_0 \epsilon_1)$. Find a left invariant Keisler measure on $G = G(\mathfrak{C})$ which is not right invariant.

**Problem 6.** (i) Let $G(\mathbb{Q}) := (\mathbb{Q}^2, +)$ seen as a group definable in $(\mathbb{Q}, +, \leq)$. Prove that $G = G(\mathfrak{C})$ has $2^{\aleph_0}$ global left invariant types.
(ii) Let $G(\mathbb{R}) := (\mathbb{R}^2, +)$ seen as a group definable in RCF. Prove that $G = G(\mathfrak{C})$ has unboundedly many global left invariant types.

**Question** Assume $p \in S_G(\mathfrak{C})$ is a strong $f$-generic. Is it true that for sufficiently large countable sublanguages $L_0$ of $L$, the type $p|_{L_0}$ is strongly $f$-generic (working in the reduct to $L_0$)?