Theories with NIP. List 3.

We work in a monster model $\mathfrak{C}$ of a complete theory T.

Problem 1. Let $M \models \mathrm{DLO}$. Let $p \in S_1(M)$ be determined by a cut of infinite cofinality on both sides. Prove that p has exactly two global invariant extensions and that they are finitely satisfiable in M.

Problem 2. Let $(a_i)_{i\in\omega} + (b_i)_{i\in\omega^*}$ be an indiscernible sequence. Prove that $(a_i)_{i\in\omega}$ is based on $(b_i)_{i\in\omega^*}$.

Problem 3. Assume T has NIP. Let I be a totally indiscernible endless sequence. Prove that $\lim(I)$ is generically stable.

Problem 4. Assume T has NIP. Let $I = (a_i)_{i\in\mathcal{I}}$ be a totally indiscernible endless sequence. Let $\mathrm{Cb}(I) := \bigcap\{\mathrm{dcl}^{eq}(\{a_i : i \in F\}) : F \text{ an infinite subset of } \mathcal{I}\}$. In the book, it is claimed that (for I indexed by ω) $\mathrm{Cb}(I)$ is the canonical base of $p := \lim(I)$ in the sense:

(i) p is definable over $\mathrm{Cb}(I)$,

(ii) for every $f \in \mathrm{Aut}(\mathfrak{C})$, $f(p) = p \iff f|_{\mathrm{Cb}(I)} = \mathrm{id}$.

Prove (i) and (ii). In case of problems with (ii), prove it for I indexed by a set of size $\geq |T|$.

It is also claimed in the book (for I indexed by ω) that

(iii) I is based on a set $A \iff$ it is indiscernible over A and $\mathrm{Cb}(I) \subseteq \mathrm{dcl}^{eq}(A)$.

Prove it. In case of problems with $\Longrightarrow$, show this implication for I indexed by a set of size $\geq |T|$.

Problem 5. Assume $\pi(x)$ is a partial (fully) stable type. Prove that every infinite indiscernible sequence of realizations of $\pi(x)$ is totally indiscernible.