Theories with NIP. List 6.

We work in a monster model $\mathfrak{C}$ of a complete theory T. Assume $\mathfrak{C}$ is infinite.

Problem 1. Let p_t, $t < \alpha$, be A-invariant types. Let I_t be a Morley sequence in p_t over $AI_{<t}$, for $t < \alpha$. Prove that the sequences I_t, $t < \alpha$, are mutually indiscernible over A.

Problem 2. Prove that $\kappa_{inp}(T) = \aleph_0$ if and only if $\kappa_{inp}(p(x)) \leq \aleph_0$ for every finitary type $p(x)$.

Problem 3. Let $p(x)$ be a partial type. Prove that the following conditions are equivalent.

(i) $\kappa_{inp}(p) > \kappa$.

(ii) There is an inp-pattern of length κ.

(iii) There is an inp-pattern of length κ witnessed by mutually indiscernible sequences $(b_i^\alpha)_{i<\omega}$, $\alpha < \kappa$.

Observe that that an analogous proof works for "ict" in place of "inp".

Problem 4. Prove that T has NIP if and only if $\kappa_{ict}(T) < \infty$.
Hint. Use Propositions 1 and 2 from page 32 from the notes.

Problem 5. Assume that T has NIP. Prove that $\kappa_{inp} = \kappa_{ict}$ (as functions on partial types).

Problem 6. Let T be the theory of an infinite set in the empty language. Prove that dp-rk$(x = x) = 1$ and $\kappa_{ict}(x = x) = 2$ (where $|x| = 1$).

Problem 7. Prove that the following conditions are equivalent for any given partial type $p(x)$.

(i) $p(x)$ is algebraic.

(ii) dp-rk$(p(x)) = 0$.

(iii) $\kappa_{ict}(p(x)) = 1$.

Problem 8. Let T be the model companion of the theory of two linear orders. Prove that dp-rk$(x = x) = 2$ (where $|x| = 1$), but there is no type of dp-rk 1.