Stable groups, List 11

Problem 1. Let D be a strongly minimal set definable over A. Let $\bar{d}$ be a finite tuple of elements of D. Prove that $RM(\bar{d}/A) = \dim_{\mathrm{acl}_A}(\bar{d})$, where $\dim_{\mathrm{acl}_A}$ is the dimension in the pregeometry on D given by the closure operator acl_A; in particular, $RM(\bar{d}/A)$ is finite.

Problem 2. (i) Prove that a simple group G of Morley rank 3 interprets an infinite field K such that $G \cong PSL_2(K)$.
(ii) Prove that if a connected group of Morley rank 3 is not solvable, then $Z(G)$ is finite and G interprets an infinite field K such that $G/Z(G) \cong PSL_2(K)$.
Hint. Use Frecon's theorem that there are no bad groups of Morley rank 3.

Problem 3. Let G and H be connected groups definable in some structure M. Assume G acts definably (e.g. trivially) on H. Prove that $H \rtimes G$ is also connected as a group definable in M.

Problem 4. Assume that in a stable structure a definable group G acts definably and faithfully on a strongly minimal set A. Let F be an infinite definable subgroup of G. Prove that there are only finitely many orbits of the action of F on A, and exactly one of them is infinite.

Problem 5. Assume that in a stable structure a definable group G acts definably on a definable set A. Define an equivalence relation $\sim$ on A by $x \sim y \iff \mathrm{Stab}_G(x)^0 = \mathrm{Stab}_G(y)^0$. Prove that $\sim$ is G-invariant.

Problem 6. Assume G is a group acting transitively on a set A. Let $a \in A$. Assume that $\mathrm{Stab}_G(a)$ acts strictly 2-transitively on $A \setminus \{a\}$. Prove that the action of G on A is strictly 3-transitive.

Problem 7. Let A be the projective line over a filed K. Let $s\colon A \to A$ be given by $s(x) := \frac{1}{x}$ and $H := \{f\colon A \to A : f(x) = ax + b \text{ for some } a \in K \setminus \{0\} \text{ and } b \in K\}$.
(i) Using Problem 5, show that $\langle H \cup \{s\}\rangle$ acts strictly 3-transitively on A.
(ii) Prove that $\langle H \cup \{s\}\rangle$ coincides with the group of all homographies $x \mapsto \frac{ax+b}{cx+d}$ (where the determinant is non-zero) of A, and that this group of homographies is isomorphic with $\mathrm{PSL}_2(K)$.

Problem 8. Let G and H be A-definable groups in an ω-stable structure. Consider $g \in G$ and $h \in H$. Prove that (g, h) is a generic of $G \times H$ over A iff h is a generic of H over A and g is a generic of G over A, h.