Stable groups, List 8

Problem 1. Prove that an infinite, ω-stable group has an infinite, definable, abelian subgroup.

Problem 2. Prove Reinecke's theorem: each minimal (in the strong sense) group is abelian.
Hint. Follow the lines of the proof of the theorem from the lecture that each minimal (in the weak sense), ω-stable group is abelian.

Problem 3. Let A be a minimal (in the week sense) group, and σ — a definable automorphism of A such that $\sigma^2 = \mathrm{id}$. Prove that $\sigma = \mathrm{id}$ or $\sigma = -\mathrm{id}$.

Problem 4. Let A be a minimal (in the strong sense, or in the weak sense assuming additionally ω-stability) group. Prove that A is either infinite, elementary abelian of prime exponent p (i.e. an infinite dimensional vector space over $\mathbb{F}_p$), or a divisible group with only finitely many elements of any given finite order.
Comment. This is a full classification of pure minimal groups (in particular, each pure group satisfying the above dichotomy is minimal).

Problem 5. Let A be a minimal, divisible group of finite Morley rank. Assume that G is an infinite definable group of automorphisms of A. Prove that G is a definable group of automorphisms of the additive group of some definable field of characteristic 0. Conclude that G is abelian.