Stable groups, List 9

Problem 1. Let E be a definable equivalence relation on a definable set X such that $RM(X/E) \geq \beta$ and for each $x \in X$, $RM([x]_E) \geq \alpha$. Prove that $RM(X) \geq \alpha + \beta$. Conclude that for a definable subgroup H of a definable group G one always has $RM(G) \geq RM(H) + RM(G/H)$.
Comment. If $RM(G) < \omega$, then $RM(G) = RM(H) + RM(G/H)$.

Problem 2. Complete the proof of Claim 5 from the proof of Cherlin's theorem, i.e. show that if $Y := \{a \in G \setminus \{e\} : a \notin B(a)\}$ is non-empty, then $RM(Y) = 2$.

Problem 3. Let G be a stable group (in fact, it is enough to assume that all definable quotients of definable subgroups of G have icc on centralizers). Then:
(i) if H is an abelian subgroup of G, then G has a definable, abelian subgroup containing H;
(ii) if H is an n-nilpotent subgroup of G, then G has a definable, n-nilpotent subgroup containing H;
(iii) if H is an n-solvable subgroup of G, then G has a definable, n-solvable subgroup containing H.

Problem 4. Let $G = \mathfrak{C}$ be an ω-stable group. Assume that G has a generic involution.
(i) Prove that the centralizer of some generic element has finite index in G.
(ii) Conclude that G is abelian-by-finite.