Stochastic networks and related topics III

Banach Center, Będlewo, Poland

22 May – 27 May, 2011

Abstracts

Organizing committee

Krzysztof Dębicki (head) Krzysztof.Debicki@math.uni.wroc.pl
Zbigniew Palmowski Zbigniew.Palmowski@math.uni.wroc.pl
Tomasz Rolski Tomasz.Rolski@math.uni.wroc.pl
Ryszard Szekli Ryszard.Szekli@math.uni.wroc.pl

Marek Arendarczyk (secretary) Marek.Arendarczyk@math.uni.wroc.pl
Paweł Lorek (webmaster) Pawel.Lorek@math.uni.wroc.pl

The organization of this conference is supported by

- Mathematical Institute, University of Wrocław
- Stefan Banach International Mathematical Center

Contents

WHAT RISKS LEAD TO RUIN

VENKAT ANANTHARAM *University of California, Berkeley*

Narayana Prasad Santhanam (University of Hawaii, Manoa)
and
Venkat Anantharam (University of California, Berkeley)

Insurance transfers losses associated with risks to the insurer for a price, the premium. Considering a natural probabilistic framework for the insurance problem, we derive a necessary and sufficient condition on nonparametric loss models such that the insurer remains solvent despite the losses taken on.

We adopt the collective risk approach. Namely, we abstract the problem to include just two agents: the insured and the insurer.

We model the loss at each time by numbers in $\mathbb{N} := \{0, 1, ...\}$. A loss distribution is a distribution over $\mathbb{N}$., Let $\mathcal{P}$ be a set of loss distributions. $\mathcal{P}^\infty$ is the collection of i.i.d. measures over infinite sequences from $\mathbb{N}$ such that the marginal distribution over $\mathbb{N}$ is in $\mathcal{P}$. Each $p \in \mathcal{P}$ is assumed to have finite support. An insurer's scheme is defined by the premium demanded by the insurer from the insured at each time as a function of the loss sequence observed up to that time. The insurer is allowed to wait for some period before beginning to insure the process, but once insurance commences, the insurer is committed to continue insuring the process. All that the insurer knows is that the loss sequence is a realization from some process with law in $\mathcal{P}^\infty$; the insurer does not know which $p \in \mathcal{P}$ describes the distribution of the loss sequence. The insurer goes bankrupt when the loss incurred exceeeds the built up buffer of reserves from premiums charged so far.

A distribution $p \in \mathcal{P}$ is called *deceptive* if $\forall \epsilon > 0\ \exists \delta > 0$ so that no matter what function $f : \mathbb{R} \mapsto \mathbb{R}$ is chosen, there is a distribution $q \in \mathcal{P}$ such that $J(p, q) \leq \epsilon$ and $F_q^{-1}(1-\delta) > f(F_p^{-1}(1-\delta))$. Here, for $p \in \mathcal{P}$, F_p denotes the linearly interpolated cumulative distribution function p, and for $p, q \in \mathcal{P}$,

$$J(p,q) := D(p\|\frac{p+q}{2}) + D(q\|\frac{p+q}{2}) ,$$

where, for probability distribution p and q on $\mathcal{N}$, $D(p\|q)$ denotes the relative entropy of p with respect to q.

We show that a collection $\mathcal{P}^\infty$ of finite support i.i.d. processes is insurable iff there are no deceptive distributions among its single letter marginals $\mathcal{P}$. Note that, even though we assume a finite range for each $p \in \mathcal{P}$, there is no absolute bound assumed on the possible loss at any time.

(Joint work with Parv Venkitasubramaniam.)

ON THE TAIL ASYMPTOTICS OF THE AREA SWEPT UNDER THE WORKLOAD GRAPH

MAREK ARENDARCZYK *Mathematical Institute, University of Wrocław, marendar@math.uni.wroc.pl*

Let $\{Q(t); t \geq 0\}$ be the stationary workload process of a fluid queue fed by a standard Brownian motion $\{B(t); t \in \mathbb{R}\}$ and emptied by a constsnt rate $c > 0$.
We analyze asymptotics of

$$\mathbb{P}\left(\int_0^{T_u} Q(t)\mathrm{d}t > u\right) := \pi(T_u, u),$$

as $u \to \infty$.
In particular

- for $T_u = o(\sqrt{u})$ we obtain exact asymptotics of $\pi(T_u, u)$;
- for $T_u = A\sqrt{u}$ we obtain logarithmic asymptotics of $\pi(T_u, u)$.

Additionally we derive Laplace transform of $\pi(\tau, u)$ for τ being the length of the running busy period of $\{Q(t)\}$.

This talk is based on joint work with Krzysztof Dębicki and Michel Mandjes.

A FUNCTIONAL LIMIT THEOREM FOR DEPENDENT REGULARLY VARYING SEQUENCES

BOJAN BASRAK *Department of Mathematics, University of Zagreb*, bbasrak@math.hr

It is well known that under an appropriate regular variation condition, the affinely normalized partial sums of a sequence of independent and identically distributed random variables converge weakly to a non-Gaussian stable random variable. A functional version of this is known to be true as well, the limit process being a stable Lévy process. It turns out that for a stationary regularly varying sequence, the properly centered partial sum process still converges to a stable Lévy process under suitable mixing condition. Due to clustering, the Lévy triple of the limit process can be different from the one in the independent case. Also, the convergence takes place in the space of càdlàg functions endowed with Skorohod's M_1 topology, and fails in the more usual J_1 topology. The result rests on a new limit theorem for point processes which is of independent interest. We explain how the theory applies to some time series models popular in applications.

References

[1] Basrak, B., Krizmanić, D. and Segers, J. (2011). A functional limit theorem for partial sums of dependent random variables with infinite variance. To appear in *Ann.Probab.* Avaliable at: http://arxiv.org/abs/1001.1345

Backward-forward extrema of spectrally one-sided Lévy processes with an application to queueing

ERIK BAURDOUX *London School of Economics*, e.j.baurdoux@lse.ac.uk

We consider a spectrally one-sided Lévy process X (i.e. jumps occur only in one direction) and study random variables of the form

$$S_T := \sup_{t \le T} \sup_{s \ge t} (X_s - X_t),$$

where $T > 0$. In a Lévy queue, this object is related to the workload during the time interval $[0, T]$. Using fluctuation theory of Lévy processes we derive expressions for the distribution of $S_{\mathbf{e}_q}$, where $\mathbf{e}_q$ denotes an exponential random variable independent of X with parameter $q > 0$. We study asymptotics of $P(S_T > x)$ as $x \to \infty$ (fixed T) and also as $T \to \infty$ (fixed x). Related problems have recently also been studied (independently) in [1].
This talk is based on joint work with Zbigniew Palmowski and Martijn Pistorius.

References

[1] K. Dębicki, K. Kosiński and M. Mandjes. (2011) On the infimum attained by a reflected Lévy process, EURANDOM Report 2011-005.

MONTE CARLO METHODS FOR STOCHASTIC NETWORKS

JOSE BLANCHET *Columbia University*, jose.blanchet@columbia.edu

We will discuss efficient Monte Carlo techniques for the analysis of stochastic networks. Our discussion involves both rare-event simulation topics and perfect sampling of stochastic networks (i.e. sampling without bias from the associated steady-state distribution). For instance, we shall explain how importance sampling techniques, which are widely used in rare-event simulation, can be utilized in the design of perfect sampling of stochastic networks of interest. We will illustrate the technique in the context of several networks of interest, such as multi-server queues and reflected Brownian motion. Although perfect sampling techniques have been known since the early nineties, virtually all of the perfect samplers known for stochastic networks have been designed either for variations of the Markovian Jackson networks (taking advantage of an associated process with product form solution) or for one dimensional processes. The algorithms that we discuss greatly relax these types of assumptions.

CELLULAR NETWORKS WITH SHADOWING

BARTŁOMIEJ BŁASZCZYSZYN *INRIA/ENS*, Bartek.Blaszczyszyn@ens.fr

Using a spatial version of the Erlang's loss formula and the Kaufman-Roberts algorithm we show how the blocking probability of a constant bit-rate traffic depends on the variance of the log-normal shadowing and on the path-loss exponent in regular, hexagonal networks. Both functions exhibit a lack of monotonicity. In order to explain these observations, we study the mean path-loss with respect to the serving base station (BS) and the mean interference factor, defined as the ratio of the sum of the path-gains form interfering BS to the path-gain from the serving BS. We also compare them to those obtained for Poisson networks. We observe, as commonly expected, that a strong variance of the shadowing increases the mean path-loss with respect to the serving BS, which in consequence increases the blocking probability. However, we also obtain a more surprising result saying that in some cases an increase of the variance of the shadowing can significantly reduce the mean interference factor and, in consequence, also the blocking probability. We confirm our findings by a mathematical analysis of the respective models. We also obtain fully explicit, analytical results for the mean path-loss and interference factors in the case of the infinite Poisson network with an arbitrary distribution of the shadowing.

References

[1] Błaszczyszyn, B. and Karray, M. How the path-loss with log-normal shadowing impacts the quality of service in cellular networks; why blocking probability is not always increasing in the shadowing variance. Tech. Rep. *http://hal.inria.fr/inria-00562462*, 2011.

WIRELESS RANDOM-ACCESS NETWORKS: FAIRNESS, PERFORMANCE AND STABILITY

SEM BORST *Eindhoven University of Technology & Alcatel-Lucent Bell Labs*,
sem@win.tue.nl

Random-access algorithms such as the Carrier-Sense Multiple-Access (CSMA) protocol provide a popular mechanism for distributed medium access control in emerging large-scale wireless networks. In the CSMA protocol, each node attempts to access the medium after a certain back-off time, but nodes that sense activity of interfering nodes freeze their back-off timer until the medium is sensed idle. Under suitable assumptions, the joint activity state of the various nodes may be modeled as a reversible Markov process with a convenient product-form stationary distribution.

The above-mentioned models tend to assume exponential back-off periods and transmission durations, and consider a scenario where buffers are saturated, and nodes always have packets pending for transmission. In reality, back-off periods and transmission times are non-exponential, while the buffers may occasionally be empty as packets are randomly generated and transmitted over time. Motivated by these observations, we examine two extensions: (i) general distributions of transmission times and back-off periods; and (ii) queueing dynamics in scenarios with packet arrivals. We show that the the stationary distribution of the joint activity state is not only insensitive with respect to the distribution of the transmission times, but also that of the back-off periods. In addition, we explicitly identify the stability conditions in a few relevant scenarios, and illustrate the difficulty arising in other cases.

Also, the stationary distribution may not reflect the performance over shorter time intervals, and in particular, long-term fairness may hide severe starvation effects over shorter time periods due to meta-stability. We will discuss how the short-term throughput performance is affected by the size and structure of the network topology.

Note: based on joint work with Niek Bouman (TU/e), Johan van Leeuwaarden (TU/e), Alexandre Proutiere (Microsoft), Patrick Thiran (EPFL) and Peter van de Ven (TU/e).

THE JOINT QUEUE LENGTH DISTRIBUTION IN POLLING SYSTEMS

ONNO BOXMA *Eurandom and Department of Mathematics and Computer Science, Eindhoven University of Technology, the Netherlands, boxma@win.tue.nl*

Streszczenie

In this paper we consider a *polling system*: a queueing system of $N \geq 1$ queues, with Poisson arrivals, visited in a cyclic order (with or without switchover times) by a single server. For this system we derive an expression for the probability generating function (PGF) of the joint queue length distribution at an arbitrary epoch in a stationary cycle, under no assumptions on service disciplines. We also derive the Laplace-Stieltjes transform (LST) of the joint workload distribution at an arbitrary epoch. The formulas for the PGF and the LST depend on V_{b_i} and V_{c_i}, the probability generating functions of the joint queue length distribution at visit beginnings and visit completions at the ith queue, respectively. These PGF's can be computed in many cases, the most important one being the branching case, when all the service disciplines satisfy a *branching property*.

Note: this is joint work with Offer Kella and Kamil Kosiński.

RUIN PROBABILITY WITH PARISIAN DELAY FOR A SPECTRALLY NEGATIVE LÉVY RISK PROCESS

IRMINA CZARNA *University of Wrocław*, czarna@math.uni.wroc.pl

In this talk we present, for a spectrally negative Lévy process, a compact formula for the Parisian ruin probability, which is defined by the probability that the process exhibits an excursion below zero which length exceeds a certain fixed period r. The formula involves only the scale function of the spectrally negative Lévy process and the distribution of the process at time r.

References

[1] Loeffen, R., Czarna, I. and Palmowski Z. (2010) Parisian ruin probability for spectrally negative Lévy processes, *Manuscript.*

[2] Czarna, I. and Palmowski Z. (2010) Ruin probability with Parisian delay for a spectrally negative Lévy risk process, *Manuscript.*

STABLE SUBNETWORKS IN NON-ERGODIC NETWORKS

HANS DADUNA *University of Hamburg, Department of Mathematics*,
daduna@math.uni-hamburg.de

We contribute to the following problem which was investigated by Goodman and Massey [1]: In large and complex networks, unstable subnetworks often occur due to local overload or due to non-availability of nodes or links, but nevertheless, there co-exist regions where other subnetworks stabilize locally. In the framework of Jackson networks Goodman and Massey proved that asymptotically the stable subnetwork looks like an ergodic Jackson network, while the queues in the unstable nodes grow unboundedly. We consider this phenomenon in networks where nodes are unreliable and are not available for some time.

The talk is on joint work with Jennifer Mylosz (University of Hamburg, Department of Mathematics).

References

[1] Goodman, J. B., Massey, W. A., The non–ergodic Jackson network,
Journal of Applied Probability 21, p. 860-869, 1984

QUEUEING OUTPUT PROCESSES REVISITED: A POINT PROCESS PERSPECTIVE

D.J. DALEY *Dept Mathematics and Statistics, The University of Melbourne*,
dndaley@gmail.com

Prompted by an 'open problem' paper by Nazarathy in QUESTA (forthcoming), the paper exploits a known integral formula for the variance of the count function $N(0, x]$ of a stationary orderly point process to study asymptotic relations for this variance. A renewal process illustrates well the behaviour: the variance is asymptotically linear when the generic lifetime r.v. X has finite second moment, and it has an asymptotically linear behaviour $B_1 x + B_0 + o(1)$ when X has a non-lattice distribution and finite third moment. Sample function relations enable quick computation of the asymptotic variance for M/M/k/K system output satisfying this linear behaviour. A conjecture for the asymptotic variance of a point process with an embedded regenerative structure is described.

STABILITY OF A MARKOV-MODULATED MARKOV CHAIN, WITH AN APPLICATION TO A WIRELESS NETWORK GOVERNED BY TWO PROTOCOLS

SERGEY FOSS *Heriot-Watt, Edinburgh and Institute of Mathematics, Novosibirsk*

We consider a discrete-time Markov chain (X^n, Y^n), where the X component forms a Markov chain itself. Assuming that (X^n) is ergodic, we formulate the following "naive" conjecture.

Consider an auxiliary Markov chain $\{\widehat{Y}^n\}$ whose transition probabilities are the averages of transition probabilities of the Y-component of the (X, Y)-chain, where the averaging is weighted by the stationary distribution of the X-component. The conjecture is: if the $\widehat{Y}$-chain is positive recurrent, then the (X, Y)-chain is positive recurrent too.

We first show that, under appropriate technical assumptions, such a general result indeed holds, and then apply it to two versions of a multi-access wireless model governed by two randomised protocols.

CONNECTIVITY OF RANDOM GEOMETRIC GRAPHS RELATED TO MINIMAL SPANNING FORESTS

CHRISTIAN HIRSCH *Institute of Stochastics, Ulm University*,
christian.hirsch@uni-ulm.de

For any locally finite set $M \subset \mathbb{R}^d$, $d \geq 2$, the minimal spanning forest $\mathrm{MSF}(M)$ is a geometric graph with vertex set M whose edge set is constructed as follows. Any two vertices $x, y \in M$ are connected if and only if there does not exist an integer $m \geq 2$ and a sequence $x_0 = x, \ldots, x_m = y \in M$ such that

$$|x_k - x_{k+1}| < |x - y| \qquad \text{for all } k = 0, \ldots, m-1\,. \tag{1}$$

Aldous and Steele [1] conjectured that $\mathrm{MSF}(X)$ is almost surely connected if $X \subset \mathbb{R}^d$ is a homogeneous Poisson point process. This conjecture was proven in [2] for dimension $d = 2$. However, it remains open for $d \geq 3$ (and for non–Poisson point processes X).

In this talk we investigate the connectivity of random geometric graphs which can be seen as approximations of minimal spanning forests. For any $n \geq 2$ and $M \subset \mathbb{R}^d$ locally finite, we consider the graph $G_n(M) \supset \mathrm{MSF}(M)$ with vertex set M, where any two vertices $x, y \in M$ are connected if and only if there does not exist an integer $m \in \{2, \ldots, n\}$ and a sequence $x_0 = x, \ldots, x_m = y \in M$ such that (1) holds. Note that

$$G_2(M) \supset G_3(M) \supset \ldots \supset \cap_{n=2}^{\infty} G_n(M) = \mathrm{MSF}(M)\,.$$

Furthermore, $G_2(M)$ is the so–called β–skeleton induced by M for $\beta = 2$.

We derive sufficient criteria for the connectivity of $G_n(X)$, which are satisfied for a large class of point processes $X \subset \mathbb{R}^d$. These criteria are related to a descending–chains condition (see [4]) and to some annulus-type continuum percolation model considered e.g. in [3]. In this way, connectivity of $G_n(X)$ can be shown not only for the point processes discussed in [4]. But, using results presented in [5], we can prove that $G_n(X)$ is almost surely connected for any stationary point processes X with finite range of dependence and absolutely continuous second factorial moment measure.

The talk is based on joint work with D. Neuhäuser and V. Schmidt.

References

[1] D. Aldous and J.M. Steele. Asymptotics for Euclidean minimal spanning trees on random points. *Probability Theory and Related Fields* **92** (1992), 247–258.
[2] K.S. Alexander. Percolation and minimal spanning forests in infinite graphs. *Annals of Probability* **23** (1995), 87–104.
[3] P. Balister, B. Bollobás and M. Walters. Continuum percolation with steps in an annulus. *Annals of Applied Probability* **14** (2004), 1869–1879.
[4] D. Daley and G. Last. Descending chains, the lilypond model, and mutual–nearest–neighbour matching. *Advances in Applied Probability* **37** (2005), 604–628.
[5] T.M. Liggett, R.H. Schonmann and A.M. Stacey. Domination by product measures. *Annals of Applied Probability* **25** (1997), 71–95.

ONE-SIDED MARKOV ADDITIVE PROCESSES: EXIT FROM AN INTERVAL AND THE SCALE MATRIX

JEVGENIJS IVANOVS *Eurandom, Eindhoven University of Technology*
Korteweg-de Vries Institute for Mathematics, University of Amsterdam, ivanovs@eurandom.tue.nl

Markov Additive Processes (MAPs) are a natural generalization of Lévy processes to regime switching models. They provide additional flexibility often required in applications; one may think of seasonality of prices, burst arrivals, occurrence of events in phases and so on. We assume that the process of interest has jumps in one direction only, which is reasonable in many applications. As shown in the recent literature on exit problems for Lévy processes, under this assumption it is often possible to arrive at substantially more explicit and transparent results. The central role in this theory is played by so called scale functions.

In this talk, for a general spectrally negative MAP, we construct its scale matrix, which is a multidimensional analogue of a scale function. This construction is considerably more challenging in the case of MAPs. It is based on some novel ideas, yielding additional insight into the problem. For example, it turns out that the scale function of a Lévy process is closely related to the expected local time at zero up to certain first passage times.

AN INSENSITIVITY OF REFLECTION MAPPINGS ON AN ORTHANT

OFFER KELLA *The Hebrew University of Jerusalem*, Offer.Kella@huji.ac.il

A reflection map on the nonnegative orthant is applied to a multidimensional process X and then to $a + X$, where a is a nonnegative constant vector. A question that has been open for over 15 years is under what conditions the difference between the two resulting processes converges to zero for any choice of a as time diverges. This in turn implies that if one imposes enough stochastic structure that ensures that the reflection map applied to X converges in distribution then it will also converge in distribution when it is applied to $\eta + X$ where η is any almost surely finite valued random vector that may even depend on the process X. In this paper we obtain a useful equivalent characterization of this property. As a result we are able to identify a natural sufficient condition in terms of the given data X and the constant routing matrix. A similar necessary condition is also indicated. Extensions of the sufficient condition are then developed for reflection maps with drift and routing coefficients that may be time and state dependent.

Joint work with **S. RAMASUBRAMANIAN**, *Indian Statistical Institute*, ram@isibang.ac.in

References

[1] Kella, O. (2006). Reflecting thoughts. *Statistics and Probability Letters* **76**, 1808-1811.

[2] Kella, O. (1997). Stochastic storage networks: stationarity and the feedforward case. *Journal of Applied Probability* **34**, 498-507.

[3] Kella, O. and W. Whitt. (1996). Stability and structural properties of stochastic fluid networks. *Journal of Applied Probability* **33**, 1169-1180.

[4] Kella, O. (1996). Stability and non-product form of stochastic fluid networks with Lévy inputs. *The Annals of Applied Probability* **6**, 186-199.

[5] Ramasubramanian, S. (2000). A subsidy-surplus model and the Skorokhod problem in an orthant. *Mathematics of Operations Research* **25**, 509-538.

TAKIS KONSTANTOPOULOS *Uppsala University*

TBA

EXTREMES OF MULTIDIMENSIONAL GAUSSIAN PROCESSES

KAMIL MARCIN KOSIŃSKI *EURANDOM, Technische Universiteit Eindhoven, the Netherlands*, kosinski@eurandom.tue.nl

This talk considers extreme values attained by a centered, multidimensional Gaussian process $X(t) = (X_1(t), \ldots, X_n(t))$ minus drift $d(t) = (d_1(t), \ldots, d_n(t))$, on an arbitrary set T. Under mild regularity conditions, we establish the asymptotics of

$$\log \mathbb{P}\left(\exists t \in T : \bigcap_{i=1}^{n} \{X_i(t) - d_i(t) > q_i u\}\right),$$

for positive thresholds $q_i > 0$, $i = 1, \ldots, n$ and $u \to \infty$. Our findings generalize and extend previously known results for the single-dimensional and two-dimensional cases. A number of examples illustrate the theory.

AN OPEN QUEUEING NETWORK WITH ASYMPTOTICALLY STABLE FLUID MODEL AND UNCONVENTIONAL HEAVY TRAFFIC BEHAVIOR

ŁUKASZ KRUK *Maria Curie - Skłodowska University, Lublin, Poland*, lkruk@hektor.umcs.lublin.pl

In this talk we provide an example of a feedforward first-in-system, first-out (FISFO) queueing network with unconventional, i.e., non-Brownian, diffusion approximation. To our knowledge, this is the first example of an open multiclass network with unconventional heavy traffic behavior. We also argue that fluid models of subcritical feedforward earliest-deadline-first (EDF) queueing networks, in particular FISFO networks, are asymptotically stable.

SAMPLE COVARIANCES FOR STOCHASTIC VOLATILITY MODELS

RAFAŁ KULIK *University of Ottawa*, rkulik at uottawa.ca

Stochastic volatility models capture one of the standardized features of financial data: the log-returns are uncorrelated, but their squares, or absolute values are (highly) dependent. Furthermore, they are flexible enough to model heavy tails. However, in case of infinite moments, asymptotic results for partial sums and sample covariances are almost non-existing. We consider the asymptotic behaviour of the partial sums and the sample autocovariances of long-memory volatility models in case of infinite moments. Depending on the interplay of assumptions on moments and the intensity of dependence, there are two types of convergence rates and limiting distributions. (Joint work with Philippe Soulier).

UNBIASED SAMPLING OF BROWNIAN MOTION

GÜNTER LAST *Karlsruhe Institute of Technology*, guenter.last@kit.edu

Let $B = (B_t)_{t \in R}$ be a two-sided standard Brownian motion. We define an *unbiased sampling* (of B) as a B-measurable (finite) random time τ such that $(B_{\tau+t} - B_\tau)_{t \in R}$ is a Brownian motion independent of B_τ. Using results of [2] and [3] we characterize unbiased samplings in terms of invariant balancing transports. In fact, such a sampling is similar to the solution of the well-known extra head problem for stationary Poisson processes, see [1]. In the second part of the talk we ask for the existence of an unbiased sampling $\tau \geq 0$ such that τ is a stopping time and B_τ has a given distribution ν. A positive answer would connect unbiased samplings with the classical Skorokhod embedding problem.

The talk is based on joint work with Peter Mörters (Bath) and Hermann Thorisson (Reykjavik).

References

[1] Holroyd, A.E. and Peres, Y.: Extra heads and invariant allocations. *Annals of Probability* **33**, 31–52 (2005).

[2] Last, G. and Thorisson, H.: Invariant transports of stationary random measures and mass-stationarity. *Annals of Probability* **37**, 790–813, (2009).

[3] Zähle, U: Self-similar random measures III. Self-similar random processes. *Math. Nachr.* **151**, 121–148, (1991).

Applying spectral representations for CBI processes to finance

RONNIE LOEFFEN *Weierstrass Institute, Berlin*

Ogura (1970) provided, under some conditions, a spectral representation of the transition kernel of (one-dimensional) continuous-state branching processes with immigration (CBI). Especially in finance, where CBI processes are known as one-dimensional, positive, affine processes, such a spectral representation is very useful, since it leads to a fast method for computing European option prices for a whole range of strikes and maturities. In this talk we revisit Ogura's result and apply it to option pricing, hereby indicating the benefits and drawbacks of the method.

ON RUIN PROBABILITIES AND PENALTY FUNCTIONS FOR DEPENDENT RISKS

STEPHANE LOISEL *ISFA, Université Lyon 1*, loisel@univ-lyon1.fr

Explicit formulas for ruin probabilities are obtained for some kinds of mixing risk models. Multivariate penalty functions are introduced and some optimal allocation problems are analyzed.

Strong Stationary Duality and Speed of Convergence to Stationarity for Möbius Monotone Markov Chains.

PAWEŁ LOREK (joint with RYSZARD SZEKLI) *University of Wrocław*,
Pawel.Lorek@math.uni.wroc.pl

For Markov chains with a partially ordered finite state space we show strong stationary duality under the condition of Möbius monotonicity of the chain. Studying aborption time in dual chain allows to obtain bounds on distance from stationarity. We illustrate general theory by an an alysis of nonsymmetric random walks on the cube with an application to networks of queues.

BIRTHDAY SURPRISES

MICHEL MANDJES *University of Amsterdam*, m.r.h.mandjes@uva.nl

The classical birthday problem is a standard exercise for our students: what is the probability that, among k people, everyone has a different birthday? Things complicate enormously, however, in the situation that the birthdays are not equally spread over the year, and also other ramifications can be thought of. In this presentation I will look at various generalized birthday problems, in various asymptotic regimes.

This talk contains joint work with Sandeep Juneja (TIFR).

FORMULA FOR THE SUPREMUM DISTRIBUTION OF A SPECTRALLY POSITIVE LÉVY PROCESS

ZBIGNIEW MICHNA

Department of Mathematics and Cybernetics, Wrocław University of Economics, Wrocław Poland
zbigniew.michna@ue.wroc.pl

We derive formula for probability $P(\sup_{t\leq T}(X(t) - ct) > u)$ where $X = \{X(t)\}$ is a spectrally positive Lévy process and $c \in \mathbb{R}$. We consider two cases: finite variation Lévy processes and infinite variation Lévy processes. As an example we investigate the α-stable Lévy process with $0 < \alpha \leq 2$ and the inverse Gaussian Lévy process.

References

[1] Z. Michna (2011) *Formula for the supremum distribution of a spectrally positive α-stable Lévy process.* Statistics and Probability Letters **81**, pp. 231–235.
[2] Z. Michna (2011) *Formula for the supremum distribution of a spectrally positive Lévy process.* Arxiv preprint arXiv:1104.1976, 2011 - arxiv.org.

REFLECTING BROWNIAN MOTION IN TWO DIMENSIONS: EXACT ASYMPTOTICS FOR THE STATIONARY DISTRIBUTION AND SOME RELATED TOPICS

MASAKIYO MIYAZAWA *Tokyo University of Science*, miyazawa@rs.tus.ac.jp

We consider a two-dimensional semimartingale reflecting Brownian motion (SRBM) in the non-negative quadrant. The data of the SRBM consists of a two-dimensional drift vector, a 2×2 positive definite covariance matrix, and a 2×2 reflection matrix. Assuming the SRBM is positive recurrent, we are interested in tail asymptotic of its marginal stationary distribution along each direction in the quadrant. For a given direction, the marginal tail distribution has the exact asymptotic of the form $bx^{\kappa} \exp(-\alpha x)$ as x goes to infinity, where α and b are positive constants and κ takes one of the values $-3/2$, $-1/2$, 0, or 1; both the decay rate α and the power κ can be computed explicitly from the given direction and the SRBM data. In particular, those values have nice geometric interpretations.

A key tool in our proof is a relationship governing the moment generating function of the two-dimensional stationary distribution and two moment generating functions of the associated one-dimensional boundary measures. This relationship allows us to characterize the convergence domain of the two-dimensional moment generating function. For a given direction c, the line in this direction intersects the boundary of the convergence domain at one point, and that point uniquely determines the decay rate α. The one-dimensional moment generating function of the marginal distribution along direction c has a singularity at α. Using analytic extension in complex analysis, we characterize the precise nature of the singularity there. Using that characterization and complex inversion techniques, we obtain the exact asymptotic of the marginal tail distribution.

The results are closely related to large deviations of the stationary distribution. We discuss how the tail asymptotics for the marginal distributions are related to the rate function of this large deviations.

This is joint work with Jim Dai, Georgia Tech.

References

[1] Dai, J. G. and Miyazawa, M. (2010) Reflecting Brownian motion in two dimensions: Exact asymptotics for the stationary distribution. Submitted for publication.

REALISABILITY PROBLEMS FOR POINT PROCESSES AND RANDOM SETS

ILYA MOLCHANOV *University of Bern*, ilya@stat.unibe.ch

The talk addresses the existence issue for a rather general random element whose distribution is only partially specified. The technique relies on the existence of a positive extension for linear functionals accompanied by additional conditions that ensure the regularity of the extension needed for interpreting it as a probability measure. It is shown in which case the extension can be chosen to possess some invariance properties.

The results are applied to obtain existence results for point processes with given correlation measure and random closed sets with given two-point covering function or contact distribution function. It is shown that the regularity condition can be efficiently checked in many cases in order to ensure that the obtained point processes are indeed locally finite and random sets have closed realisations.

RELIABILITY MODELLING WITH ON/OFF PROCESSES

ILKKA NORROS *VTT Technical Research Centre of Finland*, ilkka.norros@vtt.fi

Consider a set of k components that alternate between states 0 (on, working) and 1 (failure, downtime). We present three approaches to such systems. First, assuming stationarity, one can apply Palm calculus. Assuming the component processes to be independent, some distribution and other explicit formulas can be computed. Second, we introduce a simple model possessing 'dynamic dependence' of component failures. This is a Markov process with continuous state space, and some results have been obtained in the case of two components. Third, we define that the system is 'weakened by failures' w.r.t. some order of its path space, if its conditional distribution jumps downward at failure times and upward at repair times w.r.t. the corresponding stochastic order, and show that there are instances of the above-mentioned Markovian model that have this property. Some results on systems weakened by failures can be obtained using the theory of point process martingales.

References

[1] I. Norros, P. Kuusela, J. Lapuyade-Lahorgue, and M. Naldi. On reliability modeling with stationary on/off processes. Submitted for publication.

PERCOLATION IN HARD0-CORE SYSTEMS

MATHEW PENROSE *University of Bath*, m.d.penrose@bath.ac.uk

I shall descibe two distinct hard-core spatial models; random sequential adsorption in the square lattice, and the Poisson lilypond model in Euclidean d-space. In both cases the basic set is non-percolating but can be enhanced in a natural way, and enjoys a percolation phase transition at a non-trivial value of the enhancement parameter. Proving such results is complicated by spatial dependencies.

ON GERBER-SHIU FUNCTIONS AND OPTIMAL DIVIDEND DISTRIBUTION FOR A LEVY RISK PROCESS IN THE PRESENCE OF A PENALTY FUNCTION

M. R. PISTORIUS *Imperial College London*, m.pistorius@imperial.ac.uk

We consider an optimal dividend problem for an insurance company whose risk process evolves as a spectrally negative L´evy process (in the absence of dividend payments). The objective is to maximize the sum of the expected cumulative discounted dividends received until the moment of ruin and a penalty payment at the moment of ruin, which is an increasing function of the size of the shortfall at ruin; in addition, there may be a fixed cost for taking out dividends.

We derive an analytically explicit characterization of the optimal strategy and find an explicit necessary and sufficient condition for optimality of single band strategies, in terms of a particular Gerber-Shiu function. This is joint work with Florin Avram and Zbigniew Palmowski.

Keywords: Optimal control, L´evy process, De Finetti model, transaction costs, singular control, variational inequality, band policies, Gerber-Shiu function

References

[1] Avram, F., Palmowski, Z. and Pistorius, M.R. (2007) On the optimal dividend problem for a spectrally negative L´evy process, Ann. Appl. Probab. 17, pp. 156–180.

[2] Kyprianou, A.E., Rivero, V. and Song, R. (2008) Convexity and smoothness of scale functions and de Finettis control problem, Preprint.

[3] Loeffen, R. (2008) On optimality of the barrier strategy in de Finetti's dividend problem for spectrally negative L´evy processes, Ann. Appl. Probab. 18, pp. 1669–1680.

[4] Loeffen, R. and Renaud, J-F. (2009) De Finetti's optimal dividends problem with an affine penalty function at ruin. Insurance: Mathematics and Economics.

Counting, sampling and maximizing spanning subgraphs subject to local constraints in large sparse networks.

JUSTIN SALEZ *INRIA and Ecole Normale Superieure*

Using the theory of negative association for measures and the notion of random weak limits of sparse graphs, we establish the general validity of the cavity method for counting, maximizing and sampling spanning subgraphs subject to various kind of local constraints in asymptotically tree-like graphs. As an illustration, we provide an explicit-limit formula for the b-matching number of an Erdős-Rényi random graph with fixed average degree and diverging size, for any integer b.

WHAT IS THE EFFECT OF HEAVY TAILS IN RANDOM ENVIRONMENT?

GENNADY SAMORODNITSKY *Cornell University*

The usual weak limit theorems for sums of i.i.d. heavy tailed random variables give stable limits. The situation is very different when the heavy tails act in the random environment. (Joint work with Jonathon Peterson).

ON THE DISTRIBUTION OF TYPICAL SHORTEST–PATH LENGTHS IN CONNECTED RANDOM GEOMETRIC GRAPHS

VOLKER SCHMIDT *Institute of Stochastics, Ulm University*, volker.schmidt@uni-ulm.de

We consider random geometric graphs in $\mathbb{R}^2$ represented by their random edge set G which is assumed to be connected and stationary. Furthermore, we consider two stationary Coxian point processes X_H and X_L in $\mathbb{R}^2$ whose random intensity measures are concentrated on G. We assume that (i) X_H and X_L are conditionally independent given G and (ii) their random intensity measures are proportional to the one–dimensional Hausdorff measure ν_1 on G, i.e., $\mathbb{E}X_H(B) = \lambda_\ell\, \mathbb{E}\nu_1(B \cap G)$ and $\mathbb{E}X_L(B) = \lambda'_\ell\, \mathbb{E}\nu_1(B \cap G)$ for each Borel set $B \subset \mathbb{R}^2$ and for some (linear) intensities $\lambda_\ell, \lambda'_\ell > 0$. Each point of X_L is assumed to be connected to its closest neighbor in X_H, where two different meanings of 'closeness' are considered: either with respect to the Euclidean distance (case (e)), or in a graph–theoretic sense, i.e., along the edges of G (case (g)).

In applications, e.g. to hierarchical telecommunication networks, the edge set G can represent the underlying infrastructure, for instance, an inner–city street system. The Cox processes X_H and X_L can then describe the locations of (higher– and lower–level) network components. In this case, one is especially interested in the distribution of the typical shortest–path length C^* along the edge set between the points of X_L and their closest neighbors in X_H, which is an important performance characteristic in cost and risk analysis as well as in strategic planning of wired telecommunication.

Even for simple examples of connected and stationary geometric graphs G in $\mathbb{R}^2$, the distribution of C^* is not known analytically. However, asymptotic results can be derived if the graph G becomes unboundedly sparse and dense, respectively. In particular, we show that the limit distributions of C^* do not depend on the selected 'closeness' scenarios (e) or (g). Furthermore, for g-closeness, we show that the distribution of C^* does not depend on λ'_ℓ and decreases stochastically in λ_ℓ. It seems to be an open problem whether the latter property is also true for the case of e-closeness. On the other hand, it can be shown that under g-closeness, the distribution of C^* is stochastically smaller than under e-closeness.

Recently, in [4], the limit distributions of C^* have been determined for e-closeness, where it has been assumed that G is the edge set of a stationary Poisson–type tessellation with bounded convex cells. We extend these results performing an asymptotic analysis for typical shortest–path lengths on random geometric graphs induced by aggregate Poisson–Voronoi tessellations whose cells can be non-convex (see [2]). Furthermore, we consider β–skeletons on homogeneous Poisson point processes, which can have non–convex cells as well as dead ends (see [1]).

The talk is based on joint work with D. Neuhäuser, C. Hirsch and C. Gloaguen.

References

[1] D. Aldous and J. Shun. Connected spatial networks over random points and a route-length statistic. *Statistical Science* **25** (2010) 275–288.
[2] F. Baccelli, G. Gloaguen, and S. Zuyev. Superposition of planar Voronoi tessellations. *Stochastic Models* **16** (2000) 69–98.
[3] D. Neuhäuser, C. Hirsch, C. Gloaguen, and V. Schmidt. On the distribution of typical shortest–path lengths in connected random geometric graphs. Preprint (submitted)
[4] F. Voss, C. Gloaguen, and V. Schmidt. Scaling limits for shortest path lengths along the edges of stationary tessellations. *Advances in Applied Probability* **42** (2010) 936–952.

LOSS RATE FOR A GENERAL LÉVY PROCESS WITH DOWNWARD PERIODIC BARRIER.

PRZEMYSŁAW ŚWIĄTEK *University of Wrocław*,
Przemyslaw.Swiatek@math.uni.wroc.pl

In this talk we consider a general Lévy process X reflected at downward periodic barrier A_t and constant upper barrier K giving a process $V_t^K = X_t + L_t^A - L_t^K$. We find the expression for a loss rate defined by $l^K = E_{\pi_K} L_1^K$ and identify its asymptotics as $K \to \infty$ when X has light-tailed jumps and $EX_1 < 0$.

EXTREMES OF LOCALLY SELF-SIMILAR GAUSSIAN PROCESSES

KAMIL TABIŚ *University of Wrocław, Poland*, kamil.tabis@gmail.com

Pickands' double-sum method allows us to obtain exact asymptotics for the supremum distribution of Gaussian processes. The classical application of this method is based on the use of stationary or locally stationary structure of the analyzed process $X(\cdot)$, i.e. the assumption that

$$\mathbf{Var}(X(t) - X(s)) \;=\; \text{Const} \cdot |t-s|^{\alpha}(1+o(1)) \qquad \text{as} \quad t, s \to t^* \,,$$

for some $\alpha \in (0, 2]$, where t^* is a point at which variance function of $X(\cdot)$ attains its maximum.

In the talk we focus on the exact asymptotics of supremum distribution of Gaussian processes such that

$$\mathbf{Var}(X(t) - X(s)) \;=\; \text{Const} \cdot \mathbf{Var}\,(Y(t) - Y(s))\,(1+o(1)) \qquad \text{as} \quad t, s \to t^* \,,$$

where $Y(\cdot)$ is a self-similar Gaussian process (with not necessarily stationary increments).

We will point out difficulties of the application of the double-sum method to our problem and we present a new approach, that allows us to determine the exact asymptotics of $\mathbf{P}\left(\sup_{t\in[0,T]} X(t) > u\right)$ as $u \to \infty$. In the obtained asymptotics there appear new analogs of Pickands' constants with interesting properties. The theory will be illustrated by some examples.

References

[1] J. Pickands III, (1969) *Upcrossing probabilities for stationary Gaussian processes*, *Trans. Amer. Math. Soc.* **145**, 51–73.
[2] V. I. Piterbarg, *Asymptotic methods in the theory of Gaussian processes and fields*, Translations of Mathematical Monographs 148, AMS, Providence, 1996.
[3] K. Dębicki, K. Tabiś, (2011) *Extremes of the time-average of stationary Gaussian processes*, In revision.

CONTROLLING EGOIST BEHAVIOUR IN AN AD HOC MOBILE NETWORK

PETER TAYLOR *University of Melbourne*, p.taylor@ms.unimelb.edu.au

An ad hoc mobile network does not have any base-station infrastructure to relay calls. Unless users are close enough to be able to contact each other directly, they rely on other users to act as transit nodes. This raises the question of why a selfish or 'egoist' user would act as a transit for other users' calls. This question has received some attention in the literature and various credit transfer schemes have been proposed, see, for example, references [1] and [2]. These references, however, did not approach the problem by defining precisely what it means to control selfish behaviour.

In this talk, which presents joint work with Tony Krzesinski and Guy Latouche, I shall start from first principles by defining, in terms of a socially-optimal distribution of resources, exactly what it means for an egoist user not to act selfishly. I shall go on to derive a set of minimal constraints that can be imposed an egoist to ensure that he/she acts in the desired manner.

References

[1] L. Buttyán and J.-P. Hubaux, Stimulating cooperation in self-organizing mobile ad hoc networks, in *Mobile Networks and Applications*, **8** (2003), 579–592.

[2] J. Crowcroft, R. Gibbens, F. Kelly and S. Östring, Modelling incentives for collaboration in mobile ad hoc networks, *Performance Evaluation*, **57** (2004), 427–439.

ANALYTIC PROPERTIES OF PICKING STRATEGIES IN CAROUSELS

MARIA VLASIOU *Eindhoven University of Technology*, m.vlasiou@tue.nl

We consider a system with two carousels alternately operated by one picker, where items are stored at random. Important performance characteristics are the waiting time of the picker and the throughput of the two carousels. The waiting time of the picker satisfies an equation very similar to Lindley's equation for the waiting time in the single-server queue. We study analytically various common picking strategies for carousel systems and prove that the time needed to pick a complete order satisfies a contraction mapping, thus proving the efficiency of simple iterative numerical approximations.

Heavy Traffic of the Processor Sharing Queue via Excursion Theory

BERT ZWART *CWI*, bertz@cwi.nl

Thanks to a recent result of A. Lambert (AOP 2010), it is possible to realize one busy cycle of the queue length of the Processor-Sharing (PS) queue as the image by some functional of a Levy process. The functional involves the local time process and a random time-change. I will show how to exploit this mapping to derive the heavy traffic limit of the PS queue thanks to excursion theory.

Joint work with A. Lambert and F. Simatos.

Venkat Anantharam
University of California, Berkeley

Marek Arendarczyk
University of Wrocław

Søren Asmussen
Aarhus University

Bojan Basrak
University of Zagreb

Erik Baurdoux
London School of Economics

Jose Blanchet
Columbia University

Bartłomiej Błaszczyszyn
INRIA/ENS

Sem Borst
Eindhoven University of Technology &
Alcatel-Lucent Bell Labs

Onno Boxma
Eindhoven University of Technology

Irmina Czarna
University of Wrocław

Hans Daduna
University of Hamburg

Daryl Daley
The University of Melbourne

Krzysztof Dębicki
University of Wrocław

Serguei Foss
Heriot-Watt, Edinburgh &
Institute of Mathematics, Novosibirsk

Christian Hirsch
Ulm University

Jevgenij Ivanovs
EURANDOM, TU/e

Offer Kella
The Hebrew University of Jerusalem

Takis Konstantopoulos
Uppsala University

Kamil Kosiński
EURANDOM

Łukasz Kruk
Maria Curie - Sklodowska University, Lublin

Rafał Kulik
University of Ottawa

Günter Last
Karlsruhe Institute of Technology

Ronnie Loeffen
Weierstrass Institute, Berlin

Stephane Loisel
ISFA, Université Lyon 1

Paweł Lorek
University of Wrocław

Michel Mandjes
University of Amsterdam

Zbigniew Michna
Wrocław University of Economics

Masakiyo Miyazawa
Tokyo University of Science

Ilya Molchanov
University of Bern

Ilkka Norros
VTT Technical Research Centre of Finland

Zbigniew Palmowski
University of Wrocław

Mathew Penrose
University of Bath

Martijn Pistorius
Imperial College London

Tomasz Rolski
University of Wrocław

Justin Salez
INRIA and Ecole Normale Superieure

Gennady Samorodnitsky
Cornell University

Volker Schmidt
Ulm University

Ryszard Szekli
University of Wrocław

Przemysław Świątek
University of Wrocław

Kamil Tabiś
University of Wrocław

Peter Taylor
University of Melbourne
Maria Vlasiou
Eindhoven University of Technology

Bert Zwart
CWI