Schedule & Abstracts

Non-positively Curved Groups and Spaces
18-22 September 2017, Regensburg

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Registration</td><td></td><td></td><td></td><td></td></tr>
<tr><td>09:00 - 10:15
Belegradek I</td><td>09:00 - 10:15
McCammond II</td><td>09:00 - 10:15
Belegradek III</td><td>09:00 - 10:15
Sisto II</td><td>09:00 - 10:15
Sisto III</td></tr>
<tr><td colspan="5">10:15 - 10:45
Coffe</td></tr>
<tr><td>10:45 - 12:00
McCammond I</td><td>10:45 - 12:00
Belegradek II</td><td>10:45 - 12:00
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Lunch</td></tr>
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Braun</td><td>14:00 - 15:00
Avramidi</td><td rowspan="4">17:30 - 18:30
Guided tour</td><td>14:00 - 15:00
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Panov</td><td>15:30 - 16:30
Fukaya</td><td>15:30 - 16:30
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Horbez</td><td>17:00 - 18:00
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Poster session</td><td>19:00 - ∞
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1. Minicourses

Topology of Open Non-Positively Curved Manifolds
by Igor Belegradek (Georgia Tech)

The mini-course will give a panorama of topological properties of complete open manifolds of nonnegative sectional curvature. The exposition will be loosely based on the survey in *https://arxiv.org/abs/1306.1256* except that there will be less emphasis on the speaker's work.

Braid Groups and Non-Positive Curvature
by Jon McCammond (UC Santa Barbara)

It has long been conjectured that the braid groups are non-positively curved in the sense that they have a geometric action on some complete $CAT(0)$

space, and a promising candidate has been known for some time. For each positive integer n, a contractible n-dimensional simplicial complex with a free and cocompact n-string braid group action was constructed by Tom Brady in 2001 and a piecewise-euclidean metric was added to these complexes by Brady and myself shortly afterwards. I call this the dual braid n-complex with the orthoscheme metric. These particular complexes with this particular metric were conjectured to be $CAT(0)$ spaces for every n and this conjecture has been established when n is very small. During the three lectures of the mini-course I will describe these complexes and metrics in detail, report on some recent results related to the main conjecture, and discuss some connections to other parts of mathematics. The recent results are joint work Michael Dougherty and Stefan Witzel.

The tentative titles of the three lectures are:

1. Graphical braids and orthoschemes,
2. Boundary braids and curvature,
3. The braid arrangement and polynomials.

Acylindrically Hyperbolic Groups

by Alessandro Sisto (ETH Zurich)

Acylindrically hyperbolic groups form a very large and rather diverse class of groups that includes non-elementary (relatively) hyperbolic groups, mapping class groups, outer automorphisms of free groups, $CAT(0)$ groups with rank-one elements, and many, many others. Acylindrically hyperbolic groups turn out to share many of the "largeness" properties of non-abelian free groups, including having uncountably many quotients and infinite dimensional bounded cohomology.

In the mini-course, I will explain the definition, which involves the existence of an action with certain properties on a hyperbolic space, and I will then explain the idea behind the construction of such hyperbolic space in some of the examples. We will then discuss aspects of the geometry of acylindrically hyperbolic groups.

2. Talks

Topology of ends of non-positively curved manifolds
by Grigori Avramidi (Universität Münster)

The structure of ends of a finite volume, non-positively curved, locally symmetric manifolds M is very well understood. By work of Borel and Serre, the structure of ends is captured by the rational Tits building. This is a simplicial complex built out of the algebra of the locally symmetric space which turns out to have dimension $< \dim M/2$. In this talk, I will explain aspects of the locally symmetric situation that are true for more general finite volume non-positively curved manifolds satisfying a mild tameness assumption (there are no arbitrarily small closed geodesic loops). The main result is a half-dimensional collapse phenomenon: the homology of the thin part of the universal cover vanishes in dimension greater or equal to $\dim M/2$. One application is that any complex X homotopy eqiuvalent to M has dimension $\geq \dim M/2$. Another application is that the group cohomology with group ring coefficients of the fundamental group of M vanishes in low dimensions ($< \dim M/2$). Joint work with Tam Nguyen Phan.

L2-Betti numbers and Riemannian volume
by Sabine Braun (Karlsruher Institut für Technologie)

Gromov raised the question whether there is a universal bound for the $L2$-Betti numbers of an aspherical manifold by its simplicial volume. A positive answer would yield, in combination with Gromov's main inequality, an upper bound of $L2$-Betti numbers of an aspherical manifold by its Riemannian volume provided a lower Ricci curvature bound. While the above conjecture remains open, the implication was shown by Sauer using so-called randomization techniques.

After a short introduction to $L2$-Betti numbers I will address the randomization techniques and new developments around curvature-free versions of the main inequality for $L2$-Betti numbers.

Convex cocompactness in finitely generated groups
by Matthew Cordes (Technion)

A Kleinian group is convex cocompact if its orbit in hyperbolic 3-space is quasi-convex or, equivalently, that it acts cocompactly on the convex hull of its limit set in in hyperbolic 3-space.

Subgroup stability is a strong quasi-convexity condition in finitely generated groups which is intrinsic to the geometry of the ambient group and generalizes the classical quasi-convexity condition above. Importantly, it coincides with quasi-convexity in hyperbolic groups and the notion of convex cocompactness in mapping class groups which was developed by Farb-Mosher, Kent-Leininger, and Hamenstädt.

Using the Morse boundary, I will describe an equivalent characterization of subgroup stability which generalizes the above boundary characterization from Kleinian groups. Along the way I will discuss some known results about stable subgroups of various groups, including the mapping class group and right-angled Artin groups. The talk will include joint work with Matthew Gentry Durham and joint work with David Hume.

A coarse Cartan-Hadamard theorem with application to the coarse Baum-Connes conjecture

by Tomohiro Fukaya (Tokyo Metropolitan University)

We introduced a class of metric spaces called coarsely convex spaces, and proved that it satisfies the coarse Baum-Connes conjecture. Combined with the result of Osajda-Przytycki, it implies that systolic groups and locally finite systolic complexes satisfy the coarse Baum-Connes conjecture. Our strategy is to establish a variant of Cartan-Hadamard Theorem for coarsely convex spaces and reduce the problem to the case of open cones over their ideal boundaries. This talk is based on our preprint *arXiv:1705.05588*.

Word length asymptotics for actions of some automatic (e.g. relatively hyperbolic) groups

by Ilya Gekhtman (Yale University)

Consider any nonelementary action of a hyperbolic group G on a not necessarily proper Gromov hyperbolic space X. The action is not assumed to be discrete (for example, it could be a dense subgroup of $SL_2(\mathbf{R})$) and X is not assumed to be proper (for example it could be the curve complex, on which the mapping class group acts with pseudo-Anosov elements acting as loxodromics). We prove certain asymptotic properties for the action, including the following.

(1) With respect to the Patterson-Sullivan measure on the boundary of G, the image in X of almost every word-geodesic in G sublinearly tracks a geodesic in X.

(2) The proportion of elements in a Cayley-ball of radius R in G which act loxodromically on X converges to 1 with R.

A major tool is Cannon's theorem that hyperbolic groups admit geodesic automation. The same result hold for relatively hyperbolic groups with respect to generating sets which admit a geodesic automaton, including geometrically finite Kleinian groups, and more generally to automatic structures satisfying certain axioms related to growth tightness. We also obtain results for more general Markov processes, for example showing a *nonbacktracking* random walk on a group acting nonelementarily on a Gromov hyperbolic space hits loxodromic elements with probability 1.

This is based on completed and ongoing work with Sam Taylor and Giulio Tiozzo.

Boundary amenability of $Out(F_n)$

by Camille Horbez (Université Paris-Sud)

We prove that $Out(F_n)$ is boundary amenable, i.e. it admits a topologically amenable action on a compact Hausdorff space. This holds true more generally for $Out(G)$, where G is either a torsion-free Gromov hyperbolic group (or relatively hyperbolic with free abelian parabolics), or a right-angled Artin group. This implies that $Out(F_n)$ (and all these groups $Out(G)$) satisfies the Novikov conjecture on higher signatures. This is joint work with Mladen Bestvina and Vincent Guirardel.

On the representation variety of hyperbolic fundamental groups

by Thilo Kuessner (Universität Augsburg)

We discuss some results on the representations of surface groups and hyperbolic 3-manifold groups into special linear groups.

For 3-manifolds we discuss how the fundamental class in the Bloch group (or in group homology) can be used to distinguish components of the character variety. For example, we show that (for fixed n and any N) there are 2-bridge links whose $SL_n(\mathbf{C})$-representation variety contains more than N components all corresponding to flat bundles with trivial Chern-Simons invariant.

For surfaces we discuss how an open subset of the (non-Hausdorff) quotient of the $SL_3(\mathbf{R})$-representation variety modulo the conjugation action (namely the open set of Anosov representations) can be parametrised by some invariants coming from projective geometry and taking values in a certain non-Hausdorff space. (We conjecture that the Hausdorffification of this space yields a parametrisation of the corresponding open subset of the character variety.) As an example we will use this parametrisation to determine the topology of the space of radial Anosov representations. (The latter is part of work in progress with Sungwoon Kim.)

Line arrangements with Hirzebruch property
by Dmitri Panov (King's College London)

A line arrangement of $3n$ lines in $\mathbf{C}\mathrm{P}^2$ satisfies Hirzebruch property if each line intersects others in $n+1$ points. Hirzebruch asked if all such arrangements are related to finite complex reflection groups. In this talk I'll give a positive answer to this question in the case when the line arrangement in $\mathbf{C}\mathrm{P}^2$ is a complexification of a real line arrangement in $\mathbf{R}\mathrm{P}^2$. Namely I prove that there exist exactly four such arrangements in $\mathbf{R}\mathrm{P}^2$. The general case is still open, however in a joint work with Anton Petrunin we proved that complements so Hirzberuch arrangements in $\mathbf{C}\mathrm{P}^2$ are aspherical.

Small cancellation theory and simplicial non-positive curvature
by Tomasz Prytuła (University of Southampton)

Small cancellation theory and simplicial nonpositive curvature are two combinatorial approaches to the study of non-positively curved groups. The first theory enables one to construct numerous examples of groups, while the second one leads to many properties of "metric flavour", e.g., biautomaticity, existence of an EZ-structure, fixed point theorem. In this talk I shall briefly discuss these two theories and show how all small cancellation groups can be seen as groups of simplicial non-positive curvature. Our construction, based on ideas of D. Wise, also carries through for graphical small cancellation groups. This is joint work with Damian Osajda.

Simplicial non-positive curvature and some exotic hyperbolic grps
by Jacek Świątkowski (Uniwersytet Wrocławski)

Simplicial non-positive curvature (SNPC) is an easily checkable purely combinatorial condition for a simplicial complex. It succesfully mimicks the metric concept of non-positive curvature. Unlike small cancellation conditions, SNPC works for complexes of arbitrary dimension. A *systolic group* is a group that acts geometrically on an SNPC simplicial complex. Examples of such groups exist in arbitrary cohomological dimension.

In the talk I will explain the concept of SNPC, sketch the construction of systolic (and word hyperbolic) groups in arbitrary dimension, describe various exotic properties of such groups, and comment on some other applications of the presented concepts and techniques.