

MATHEMATICAL ANALYSIS

PROBLEMS LIST 6

6.11.08

(1) Find the radius of convergence of the power series:

$$\begin{aligned}
 & \text{(a)} \sum_{n=1}^{\infty} \frac{\binom{3n}{n} x^n}{n^2}, \quad \text{(b)} \sum_{n=1}^{\infty} \frac{2^{n+7} x^{6n}}{\sqrt{n}}, \quad \text{(c)} \sum_{n=1}^{\infty} \frac{(54n+1)^n x^{3n}}{(81n+2)^n}, \\
 & \text{(d)} \sum_{n=1}^{\infty} 10^{n^2} x^{n^3}, \quad \text{(e)} \sum_{n=1}^{\infty} n! x^{2^n}, \quad \text{(f)} \sum_{n=1}^{\infty} \frac{10^n x^n}{n^{10}}, \\
 & \text{(g)} \sum_{n=1}^{\infty} \frac{x^n}{n 10^{n-1}}, \quad \text{(h)} \sum_{n=1}^{\infty} 50^n x^{2n+5}, \quad \text{(i)} \sum_{n=1}^{\infty} \frac{x^n}{n(n+1)}, \\
 & \text{(j)} \sum_{n=1}^{\infty} \frac{x^{2n}}{\sqrt{n^2+n-n}}, \quad \text{(k)} \sum_{n=1}^{\infty} \frac{4^{n+5} x^{3n+7}}{n 6^{2n}}, \quad \text{(l)} \sum_{n=1}^{\infty} \frac{(2n)! x^n}{(n!)^3}, \\
 & \text{(m)} \sum_{n=1}^{\infty} \frac{n!}{n^n} x^{n+7}, \quad \text{(o)} \sum_{n=1}^{\infty} \binom{4n}{n} x^n, \quad \text{(p)} \sum_{n=1}^{\infty} n! x^{n^2}, \\
 & \text{(q)} \sum_{n=1}^{\infty} \binom{n+10}{n} x^n, \quad \text{(r)} \sum_{n=1}^{\infty} \frac{n!(3n)!}{(2n)!(2n)!} x^n,
 \end{aligned}$$

(2) Find the limits:

$$\begin{aligned}
 & \text{(a)} \lim_{x \rightarrow 7} \left(\frac{1}{x-7} - \frac{8}{x^2-6x-7} \right), \quad \text{(b)} \lim_{x \rightarrow 0} x \sin \frac{1}{x}, \\
 & \text{(c)} \lim_{x \rightarrow 4} \frac{\sqrt{x}-2}{x-4}, \quad \text{(d)} \lim_{x \rightarrow 3} \frac{x-3}{x+2}, \quad \text{(e)} \lim_{x \rightarrow 5} \frac{x^2-6x+5}{x-5}, \\
 & \text{(f)} \lim_{x \rightarrow 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right), \quad \text{(g)} \lim_{x \rightarrow 1} \frac{x^{2007}-1}{x^{10}-1}, \\
 & \text{(h)} \lim_{x \rightarrow 1/2} \frac{8x^3-1}{6x^2-5x+1}, \quad \text{(i)} \lim_{x \rightarrow -2} \frac{x^3+3x^2+2x}{x^2-x-6}, \\
 & \text{(j)} \lim_{x \rightarrow 0} \frac{x-\sqrt{x}}{\sqrt{x}}, \quad \text{(k)} \lim_{x \rightarrow 1} \frac{(x-1)\sqrt{2-x}}{x^2-1},
 \end{aligned}$$