MATHEMATICAL ANALYSIS

PROBLEMS LIST 7

13.11.08

(1) Sketch the graph of the function $f(x)$ given by the formula ($[\dots]$ denotes the integer part, and $\{\dots\}$ denotes the fractional part):

(a) $f(x) = |x^2 - 1| - |x^2 - 4|$, (b) $f(x) = |x^2 - 8x + 15|$,

(c) $f(x) = x^2 + x + 2 - |x^2 - x - 2|$, (d) $f(x) = \{\cos x\}$,

(e) $f(x) = [\frac{4}{\pi} \arctan x]$, (f) $f(x) = 2\{\sin x\} - \{2 \sin x\}$.

(2) Solve the following equations and inequalities:

(a) $\sin x \geq \frac{1}{2}$, (b) $|\cos x| \leq \frac{\sqrt{2}}{2}$,

(c) $[\sin x] = 0$, (d) $\{\cos x\} = \frac{1}{2}$,

(e) $\{\frac{4}{\pi} \arctan x\} = 0$, (f) $\{\frac{3}{\pi} \arctan x\} \leq \frac{1}{2}$,

(g) $(x^2 - 4) \cdot \cos x \geq 0$, (h) $\left(\frac{3}{2} + \sin x\right)^{\sin x} = 1$.

(3) Compute the limits:

(a) $\lim\limits_{x \to +\infty} \dfrac{x - \sqrt{x}}{x + \sqrt{x}}$, (b) $\lim\limits_{x \to -\infty} \dfrac{x}{\sqrt{x^2 + 1}}$, (c) $\lim\limits_{x \to 0+} \dfrac{\log x}{1 + \log x}$,

(d) $\lim\limits_{x \to 0+} \dfrac{2^{1/x} + 1}{2^{-1/x} - 1}$, (e) $\lim\limits_{x \to 0-} \dfrac{2^{1/x} + 1}{2^{-1/x} - 1}$, (f) $\lim\limits_{x \to +\infty} \dfrac{2^{1/x} - 1}{2^{-1/x} + 1}$.

(4) Determine the domain of the function $f(x)$, and verify at which points it is continuous and at which discontinuous ($\operatorname{sgn} x$ is the sign of x: for $x > 0$ $\operatorname{sgn} x = 1$, for $x < 0$ $\operatorname{sgn} x = -1$, and for $x = 0$ $\operatorname{sgn} x = 0$):

(a) $f(x) = \operatorname{sgn}(\sin x)$, (b) $f(x) = \{x\} - (\{x\})^2$,

(c) $$f(x) = \begin{cases} 0 & : \quad x < 0 \\ x & : \quad 0 \leq x < 1 \\ -x^2 + 4x - 2 & : \quad 1 \leq x < 3 \\ 4 - x & : \quad x \geq 3, \end{cases}$$

(d) $$f(x) = \begin{cases} x & : \quad x \neq 2 \\ \operatorname{sgn} x & : \quad x = 2, \end{cases}$$

(e) $f(x) = \dfrac{x^3 - 1}{x^2 - 1}$, (f) $f(x) = \operatorname{sgn}(x^3 - x)$,

(g) $f(x) = [x] - [\sqrt[3]{x}]$, (h) $f(x) = x^3 \operatorname{sgn}(x)$,

(i) $f(x) = \dfrac{1}{\sqrt{x^2 + 4x + 4} + 1}$, (j) $f(x) = [x^2]$,

(k) $f(x) = \{\log_2 x\}$, (l) $f(x) = \dfrac{1}{\{x\}}$,

(m) $f(x) = \left| \left[x + \frac{1}{2}\right] - x \right|$.

(5) For which values of the parameters a and b the function $f(x)$ is continuous? Sketch the graph of $f(x)$ for such a and b.

(a) $$f(x) = \begin{cases} ax + b & : \quad x < 1 \\ x^2 & : \quad 1 \leq x < 2 \\ ax - b & : \quad 2 \leq x. \end{cases}$$

(b) $$f(x) = \begin{cases} x & : \quad x < 1 \\ x^2 + ax + b & : \quad 1 \leq x < 2 \\ x + 3 & : \quad 2 \leq x. \end{cases}$$