

MATHEMATICAL ANALYSIS

PROBLEMS LIST 10

4.12.08

- (1) Find the radius of convergence of the Maclaurin series of the functions:

(a) $f(x) = \sqrt{x+2}$, (b) $f(x) = \frac{1}{x+3}$, (c) $f(x) = \log(x+e)$.

- (2) Find the inflection points and intervals of convexity of functions given by the formulas:

(a) $x^3 + 2x^2 + 3x + 4$, (b) $x^8 - x^2 + 7x - 15$, (c) e^{-x^2} ,
(d) $\sin^4 x$, (e) $\sqrt{x} - \log x$, (f) $x^4 + \sqrt[4]{x}$.

- (3) Find the point of intersection of the tangent at the point $(2, 4)$ to the graph of the function $f(x) = x^2$ with the OY axis.

- (4) Find the point of intersection of the tangent at the point $(0, 1)$ to the graph of the function $f(x) = e^x$ with the OX axis.

- (5) Find the point of intersection of the tangents to the graph of $f(x) = x^3$ respectively at points $(-1, -1)$ and $(2, 8)$.

- (6) Compute $\int f(x) dx$ for $f(x)$ given by the formula:

(a) 10^x , (b) $\sqrt[n]{n}$, $m, n \in \mathbf{N}$, (c) $a^x e^x$, $a > 0$,
(d) $3, 4x^{-0,17}$, (e) $1 - 2x$, (f) $\left(\frac{1-x}{x}\right)^2$,
(g) $(\sqrt{x} + 1)(x - \sqrt{x} + 1)$, (h) $\frac{\sqrt{x} - x^3 e^x + x^2}{x^3}$,
(i) $(x+1)^{22}$, (j) $\frac{x^{100} - 1}{x - 1}$, (k) $\sin^2 x$,
(l) $\frac{x^3}{x+1}$, (m) $\frac{x \sqrt[6]{x} + \sqrt[7]{x}}{x^2}$.

- (7) Find a function $F(x)$, such that $F''(x)$ is equal to:

(a) $x^2 + 2x$, (b) $\cos x$, (c) e^{7x} .

- (8) Find a function $F(x)$, such that:

(a) $F''(x) = x^2 + 1$, $F'(0) = 2$, $F(0) = 3$,

- (b) $F''(x) = \frac{1}{x^3}$, $F'(2) = 1$, $F(3) = 5$,
(c) $F'''(x) = \sin x$, $F''(0) = F'(0) = F(0) = 0$,
(d) $F''(x) = \frac{1}{x^2}$, $F'(1) = F'(-1) = 1$, $F(1) = F(-1) = 3$.
- (9) Compute $\int f(x) dx$ for $f(x)$ given by the formula:
- (a) $x \sin 2x$, (b) $x e^{-x}$, (c) $x^n \log x$, $n \in \mathbf{N}$,
(d) $x^3 e^{5x}$, (e) $e^x \sin^2 x$, (f) $x 3^x$,
(g) $x \sin x \cos x$, (h) $e^{3x} \sin 2x$, (i) $\sqrt{e^x - 1}$,
(j) $e^x \sin e^x$, (k) $x e^{x^2}$, (l) $1 \cdot \sin \log x$,
(m) $e^{-x^2} x$, (n) $\frac{\cos \sqrt{x}}{\sqrt{x}}$, (o) $e^{\sqrt[3]{x}}$,
(p) $\frac{1}{x \log x \log \log x}$, (q) $\cos x e^{\sin x}$, (r) 6^{1-x} ,
(s) $\sin^5 x \cos x$, (t) $\tan x$, (u) $x e^{x^2} (x^2 + 1)$,
(v) $e^{5x} \sin 3x$, (w) $e^{5x} \cos 3x$, (x) $\sin 3x \cdot \sin 5x$,
(y) $\sin 15x \cdot e^{-4x}$, (z) $\frac{\arctan x}{x^2 + 1}$, (aa) $\frac{\arctan^7 x + 9 \arctan^5 x}{x^2 + 1}$,
(ab) $\frac{x^3}{(x-1)^{12}}$, (ac) $\frac{\log^7 x + \log^2 x}{x}$, (ad) $e^{-x^2} x^5$,
(ae) $\sin \sqrt{x}$, (af) $\frac{\sqrt{2 + \log x}}{x}$, (ag) $\frac{e^{2x}}{\sqrt[4]{e^x + 1}}$,
(ah) $\frac{1}{\sqrt{1 - x^2}}$.