

ANALIZA MATEMATYCZNA

LISTA ZADAŃ 12

18.12.08

- (1) Podaj wzór na $C_n = \sum_{i=1}^n \frac{b-a}{n} f\left(a + i \frac{b-a}{n}\right)$, a następnie oblicz

$$\lim_{n \rightarrow \infty} C_n$$

- (a) $f(x) = 1$, $a = 5$, $b = 8$, (b) $f(x) = x$, $a = 0$, $b = 1$,
 (c) $f(x) = x$, $a = 1$, $b = 5$, (d) $f(x) = x^2$, $a = 0$, $b = 5$,
 (e) $f(x) = x^3$, $a = 0$, $b = 1$, (f) $f(x) = 2x+5$, $a = -3$, $b = 4$,
 (g) $f(x) = x^2 + 1$, $a = -1$, $b = 2$,
 (h) $f(x) = x^3+x$, $a = 0$, $b = 4$, (i) $f(x) = e^x$, $a = 0$, $b = 1$.

- (2) Oblicz następujące całki oznaczone poprzez konstrukcję ciągu podziałów przedziału, odpowiadającego mu ciągu sum Riemanna, oraz jego granicy

- (a) $\int_2^4 x^{10} dx$, $(t_i = 2 \cdot 2^{i/n})$, (b) $\int_1^e \frac{\log x}{x} dx$, $(t_i = e^{i/n})$,
 (c) $\int_0^{20} x dx$, (d) $\int_1^{10} e^{2x} dx$,
 (e) $\int_0^1 \sqrt[3]{x} dx$, $(t_i = \frac{i^3}{n^3})$, (f) $\int_{-1}^1 |x| dx$,
 (g) $\int_1^2 \frac{dx}{x} dx$, $(t_i = 2^{i/n})$, (h) $\int_0^4 \sqrt{x} dx$, $(t_i = \frac{4i^2}{n^2})$.

- (3) Oblicz całki oznaczone

- (a) $\int_{-\pi}^{\pi} \sin x^{2007} dx$, (b) $\int_0^2 \arctan([x]) dx$,
 (c) $\int_0^2 [\cos(x^2)] dx$, (d) $\int_0^1 \sqrt{1+x} dx$,
 (e) $\int_{-2}^{-1} \frac{1}{(11+5x)^3} dx$, (f) $\int_{-13}^2 \frac{1}{\sqrt[5]{(3-x)^4}} dx$,
 (g) $\int_0^1 \frac{x}{(x^2+1)^2} dx$, (h) $\int_0^3 \operatorname{sgn}(x^3-x) dx$,
 (i) $\int_0^1 x e^{-x} dx$, (j) $\int_0^{\pi/2} x \cos x dx$,

$$\begin{aligned}
& \text{(k)} \int_0^{e-1} \log(x+1) dx, & \text{(l)} \int_0^\pi x^3 \sin x dx, \\
& \text{(m)} \int_4^9 \frac{\sqrt{x}}{\sqrt{x}-1} dx, & \text{(n)} \int_1^{e^3} \frac{1}{x\sqrt{1+\log x}} dx, \\
& \text{(o)} \int_1^2 \frac{1}{x+x^3} dx, & \text{(p)} \int_0^2 \frac{1}{\sqrt{x+1} + \sqrt{(x+1)^3}} dx, \\
& \text{(q)} \int_0^5 |x^2 - 5x + 6| dx, & \text{(r)} \int_0^1 \frac{e^x}{e^x - e^{-x}} dx, \\
& \text{(s)} \int_1^2 x \log_2 x dx, & \text{(t)} \int_0^{\sqrt{7}} \frac{x^3}{\sqrt[3]{1+x^2}} dx, \\
& \text{(u)} \int_0^{6\pi} |\sin x| dx, & \text{(w)} \int_0^{\pi/2} \cos x \sin^{11} x dx, \\
& \text{(x)} \int_0^{\log 5} \frac{e^x \sqrt{e^x - 1}}{e^x + 5} dx, & \text{(y)} \int_{-\pi}^\pi x^{2007} \cos x dx, \\
& \text{(z)} \int_0^{2\pi} (x - \pi)^{2007} \cos x dx.
\end{aligned}$$

(4) Udowodnij następujące oszacowania

$$\begin{aligned}
& \text{(a)} \int_0^{\pi/2} \frac{\sin x}{x} dx < 2, & \text{(b)} \frac{1}{5} < \int_1^2 \frac{1}{x^2+1} dx < \frac{1}{2}, \\
& \text{(c)} \frac{1}{11} < \int_9^{10} \frac{1}{x+\sin x} dx < \frac{1}{8}, & \text{(d)} \int_{-1}^2 \frac{|x|}{x^2+1} dx < \frac{3}{2}, \\
& \text{(e)} \int_0^1 x(1-x^{99+x}) dx < \frac{1}{2}, & \text{(f)} 2\sqrt{2} < \int_2^4 x^{1/x} dx, \\
& \text{(g)} 5 < \int_1^3 x^x dx < 31, & \text{(h)} \int_1^2 \frac{1}{x} dx < \frac{3}{4}.
\end{aligned}$$

(5) Oblicz następujące granice

$$\begin{aligned}
& \text{(a)} \lim_{n \rightarrow \infty} \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \cdots + \frac{1}{2n}, \\
& \text{(b)} \lim_{n \rightarrow \infty} \frac{1^{20} + 2^{20} + 3^{20} + \cdots + n^{20}}{n^{21}}, \\
& \text{(c)} \lim_{n \rightarrow \infty} \left(\frac{1}{n^2} + \frac{1}{(n+1)^2} + \frac{1}{(n+1)^2} + \frac{1}{(n+3)^2} + \cdots + \frac{1}{(2n)^2} \right) \cdot n, \\
& \text{(d)} \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}\sqrt{2n}} + \frac{1}{\sqrt{n}\sqrt{2n+1}} + \frac{1}{\sqrt{n}\sqrt{2n+2}} + \frac{1}{\sqrt{n}\sqrt{2n+3}} + \cdots + \frac{1}{\sqrt{n}\sqrt{3n}}, \\
& \text{(e)} \lim_{n \rightarrow \infty} \left(\sin \frac{1}{n} + \sin \frac{2}{n} + \sin \frac{3}{n} + \cdots + \sin \frac{n}{n} \right) \cdot \frac{1}{n}, \\
& \text{(f)} \lim_{n \rightarrow \infty} \left(\sqrt{4n} + \sqrt{4n+1} + \sqrt{4n+2} + \cdots + \sqrt{5n} \right) \cdot \frac{1}{n\sqrt{n}}, \\
& \text{(g)} \lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt[3]{n}} + \frac{1}{\sqrt[3]{n+1}} + \frac{1}{\sqrt[3]{n+2}} + \cdots + \frac{1}{\sqrt[3]{8n}} \right) \cdot \frac{1}{\sqrt[3]{n^2}}, \\
& \text{(h)} \lim_{n \rightarrow \infty} \frac{\sqrt[6]{n} \cdot (\sqrt[3]{n} + \sqrt[3]{n+1} + \sqrt[3]{n+2} + \cdots + \sqrt[3]{2n})}{\sqrt{n} + \sqrt{n+1} + \sqrt{n+2} + \cdots + \sqrt{2n}}, \\
& \text{(i)} \lim_{n \rightarrow \infty} \frac{n}{n^2} + \frac{n}{n^2+1} + \frac{n}{n^2+4} + \frac{n}{n^2+9} + \frac{n}{n^2+16} + \cdots + \frac{n}{n^2+n^2}, \\
& \text{(j)} \lim_{n \rightarrow \infty} \frac{4}{5n} + \frac{4}{5n+3} + \frac{4}{5n+6} + \frac{4}{5n+9} + \cdots + \frac{4}{26n},
\end{aligned}$$

- (k) $\lim_{n \rightarrow \infty} \frac{1}{7n} + \frac{1}{7n+2} + \frac{1}{7n+4} + \frac{1}{7n+6} + \cdots + \frac{1}{9n},$
- (l) $\lim_{n \rightarrow \infty} \frac{1}{7n^2} + \frac{1}{7n^2+1} + \frac{1}{7n^2+2} + \frac{1}{7n^2+3} + \cdots + \frac{1}{8n^2},$
- (m) $\lim_{n \rightarrow \infty} \frac{1}{n} \left(e\sqrt{\frac{1}{n}} + e\sqrt{\frac{2}{n}} + e\sqrt{\frac{3}{n}} + \cdots + e\sqrt{\frac{n}{n}} \right),$
- (n) $\lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n+3}} + \frac{1}{\sqrt{n+6}} + \frac{1}{\sqrt{n+9}} + \cdots + \frac{1}{\sqrt{7n}} \right) \frac{1}{\sqrt{n}},$
- (o) $\lim_{n \rightarrow \infty} \frac{n^2+0}{(3n)^3} + \frac{n^2+1}{(3n+1)^3} + \frac{n^2+2}{(3n+2)^3} + \frac{n^2+3}{(3n+3)^3} + \cdots + \frac{n^2+n}{(4n)^3},$
- (p) $\lim_{n \rightarrow \infty} \frac{n}{2n^2} + \frac{n}{2(n+1)^2} + \frac{n}{2(n+2)^2} + \frac{n}{2(n+3)^2} + \cdots + \frac{n}{50n^2},$
- (r) $\lim_{n \rightarrow \infty} \frac{n}{2n^2} + \frac{n}{n^2+(n+1)^2} + \frac{n}{n^2+(n+2)^2} + \frac{n}{n^2+(n+3)^2} + \cdots + \frac{n}{50n^2}.$