

MATHEMATICAL ANALYSIS

PROBLEMS LIST 7

23.11.09

- (1) Sketch the graph of the function $f(x)$ given by the formula
 ([...] denotes the integer part, and {...} denotes the fractional part):
- (a) $f(x) = |x^2 - 1| - |x^2 - 4|$, (b) $f(x) = |x^2 - 8x + 15|$,
 (c) $f(x) = x^2 + x + 2 - |x^2 - x - 2|$, (d) $f(x) = \{\cos x\}$,
 (e) $f(x) = [\frac{4}{\pi} \arctan x]$, (f) $f(x) = 2\{\sin x\} - \{2 \sin x\}$.
- (2) Solve the following equations and inequalities:
- (a) $\sin x \geq \frac{1}{2}$, (b) $|\cos x| \leq \frac{\sqrt{2}}{2}$,
 (c) $[\sin x] = 0$, (d) $\{\cos x\} = \frac{1}{2}$,
 (e) $\{\frac{4}{\pi} \arctan x\} = 0$, (f) $\{\frac{3}{\pi} \arctan x\} \leq \frac{1}{2}$,
 (g) $(x^2 - 4) \cdot \cos x \geq 0$, (h) $(\frac{3}{2} + \sin x)^{\sin x} = 1$.
- (3) Compute the limits:
- (a) $\lim_{x \rightarrow +\infty} \frac{x - \sqrt{x}}{x + \sqrt{x}}$, (b) $\lim_{x \rightarrow -\infty} \frac{x}{\sqrt{x^2 + 1}}$, (c) $\lim_{x \rightarrow 0+} \frac{\log x}{1 + \log x}$,
 (d) $\lim_{x \rightarrow 0+} \frac{2^{1/x} + 1}{2^{-1/x} - 1}$, (e) $\lim_{x \rightarrow 0-} \frac{2^{1/x} + 1}{2^{-1/x} - 1}$, (f) $\lim_{x \rightarrow +\infty} \frac{2^{1/x} - 1}{2^{-1/x} + 1}$.
- (4) Determine the domain of the function $f(x)$, and verify at which points it is continuous and at which discontinuous (sgn x is the sign of x : for $x > 0$ sgn $x = 1$, for $x < 0$ sgn $x = -1$, and for $x = 0$ sgn $x = 0$):
- (a) $f(x) = \operatorname{sgn}(\sin x)$, (b) $f(x) = \{x\} - (\{x\})^2$,
 (c) $f(x) = \begin{cases} 0 & : x < 0 \\ x & : 0 \leq x < 1 \\ -x^2 + 4x - 2 & : 1 \leq x < 3 \\ 4 - x & : x \geq 3, \end{cases}$ (d) $f(x) = \begin{cases} x & : x \neq 2 \\ \operatorname{sgn} x & : x = 2, \end{cases}$
 (e) $f(x) = \frac{x^3 - 1}{x^2 - 1}$, (f) $f(x) = \operatorname{sgn}(x^3 - x)$,
 (g) $f(x) = [x] - [\sqrt[3]{x}]$, (h) $f(x) = x^3 \operatorname{sgn}(x)$,
 (i) $f(x) = \frac{1}{\sqrt{x^2 + 4x + 4} + 1}$, (j) $f(x) = [x^2]$,
 (k) $f(x) = \{\log_2 x\}$, (l) $f(x) = \frac{1}{\{x\}}$,
 (m) $f(x) = \left| \left[x + \frac{1}{2} \right] - x \right|$.
- (5) For which values of the parameters a and b the function $f(x)$ is continuous? Sketch the graph of $f(x)$ for such a and b .
- (a) $f(x) = \begin{cases} ax + b & : x < 1 \\ x^2 & : 1 \leq x < 2 \\ ax - b & : 2 \leq x. \end{cases}$ (b) $f(x) = \begin{cases} x & : x < 1 \\ x^2 + ax + b & : 1 \leq x < 2 \\ x + 3 & : 2 \leq x. \end{cases}$