

MATHEMATICAL ANALYSIS

PROBLEMS LIST 10

14.12.09

- (1) Find the radius of convergence of the Maclaurin series of functions:

(a) $f(x) = \sqrt{x+2}$, (b) $f(x) = \frac{1}{x+3}$, (c) $f(x) = \log(x+e)$.

- (2) Find the inflection points and intervals of convexity of functions given by the formulas:

(a) $x^3 + 2x^2 + 3x + 4$, (b) $x^8 - x^2 + 7x - 15$, (c) e^{-x^2} ,
 (d) $\sin^4(x)$, (e) $\sqrt{x} - \log(x)$, (f) $x^4 + \sqrt[4]{x}$.

- (3) Find the point of intersection of the tangent at the point $(2, 4)$ to the graph of the function $f(x) = x^2$ with the OY axis.

- (4) Find the point of intersection of the tangent at the point $(0, 1)$ to the graph of the function $f(x) = e^x$ with the OX axis.

- (5) Find the point of intersection of the tangents to the graph of $f(x) = x^3$ respectively at points $(-1, -1)$ and $(2, 8)$.

- (6) Compute $\int f(x) dx$ for $f(x)$ given by the formula:

(a) 10^x , (b) $\sqrt[n]{n}$, $m, n \in \mathbf{N}$, (c) $a^x e^x$, $a > 0$,
 (d) $3, 4x^{-0,17}$, (e) $1 - 2x$, (f) $\left(\frac{1-x}{x}\right)^2$,
 (g) $(\sqrt{x} + 1)(x - \sqrt{x} + 1)$, (h) $\frac{\sqrt{x} - x^3 e^x + x^2}{x^3}$, (i) $(x+1)^{22}$,
 (j) $\frac{x^{100} - 1}{x - 1}$, (k) $\sin^2(x)$, (l) $\frac{x^3}{x+1}$,
 (m) $\frac{x \sqrt[6]{x} + \sqrt[7]{x}}{x^2}$.

- (7) Find a function F , such that $F''(x)$ is equal to:

(a) $x^2 + 2x$, (b) $\cos(x)$, (c) e^{7x} .

- (8) Find a function F , such that:

(a) $F''(x) = x^2 + 1$, $F'(0) = 2$, $F(0) = 3$,
 (b) $F''(x) = \frac{1}{x^3}$, $F'(2) = 1$, $F(3) = 5$,
 (c) $F'''(x) = \sin(x)$, $F''(0) = F'(0) = F(0) = 0$,
 (d) $F''(x) = \frac{1}{x^2}$, $F'(1) = F'(-1) = 1$, $F(1) = F(-1) = 3$.

- (9) Compute $\int f(x) dx$ for f given by the formula:

(a) $x \sin(2x)$, (b) $x e^{-x}$, (c) $x^n \log(x)$, $n \in \mathbf{N}$,
 (d) $x^3 e^{5x}$, (e) $e^x \sin^2(x)$, (f) $x 3^x$,
 (g) $x \sin(x) \cos(x)$, (h) $e^{3x} \sin(2x)$, (i) $\sqrt{e^x - 1}$,
 (j) $e^x \sin(e^x)$, (k) $x e^{x^2}$, (l) $1 \cdot \sin(\log(x))$,

(m)	$e^{-x^2} x,$	(n)	$\frac{\cos(\sqrt{x})}{\sqrt{x}},$	(o)	$e^{\sqrt[3]{x}},$
(p)	$\frac{1}{x \log(x) \log(\log(x))},$	(q)	$\cos(x) e^{\sin(x)},$	(r)	$6^{1-x},$
(s)	$\sin^5(x) \cos(x),$	(t)	$\tan(x),$	(u)	$x e^{x^2} (x^2 + 1),$
(v)	$e^{5x} \sin(3x),$	(w)	$e^{5x} \cos(3x),$	(x)	$\sin(3x) \cdot \sin(5x),$
(y)	$\sin(15x) \cdot e^{-4x},$	(z)	$\frac{\arctan(x)}{x^2 + 1},$	(aa)	$\frac{\arctan^7(x) + 9 \arctan^5(x)}{x^2 + 1},$
(ab)	$\frac{x^3}{(x-1)^{12}},$	(ac)	$\frac{\log^7(x) + \log^2(x)}{x},$	(ad)	$e^{-x^2} x^5,$
(ae)	$\sin(\sqrt{x}),$	(af)	$\frac{\sqrt{2 + \log(x)}}{x},$	(ag)	$\frac{e^{2x}}{\sqrt[4]{e^x + 1}}.$