MATHEMATICAL ANALYSIS

PROBLEMS LIST 13

11.01.10

(1) Compute the area of the region bounded by the given curves:
(a) $y = x^2$ and $y = 2x + 5$,
(b) $y = e^x$ and the straight line passing through points $(0, 1)$ and $(1, e)$,
(c) $y = \sin(x)$ and $y = \dfrac{2x}{\pi}$,
(d) $y = x^4$ and $y = x^3$,
(e) $y = \dfrac{1}{x}$ and $y = \dfrac{5}{2} - x$,
(f) $y = \dfrac{1}{x^2}$, $y = \dfrac{1}{x^3}$ and $x = 2$.

(2) Compute the length of the curve $y = f(x)$, $a \le x \le b$ for given $f(x)$, a and b:
(a) $x,\ 1,\ 2$, (b) $2x-3,\ -7,\ 12$, (c) $e^x,\ 1,\ 2$, (d) $\sqrt{x^3},\ 6,\ 10$, (e) $\dfrac{e^x + e^{-x}}{2},\ 0,\ 1$.

(3) For given $f(x)$, a and b compute the area of the surface obtained by rotating the curve $y = f(x)$, $a \le x \le b$ around the OX axis:
(a) $x^3,\ 0,\ 5$, (b) $e^{-x},\ 0,\ 10$, (c) $\sqrt{x},\ 0,\ 4$, (d) $\sin(x),\ 0,\ \pi$, (e) $\cos(7x),\ 0,\ 2\pi$.

(4) For given $f(x)$, a and b compute the volume of the solid obtained by rotating the region $0 \le y \le f(x)$, $a \le x \le b$ around the OX axis:
(a) $\sqrt{x},\ 0,\ 1$, (b) $x,\ 1,\ 5$, (c) $x^7,\ 0,\ 10$, (d) $e^x,\ -3,\ 0$, (e) $\sin(x),\ 0,\ \dfrac{3\pi}{2}$.

(5) Compute the length of the curve $y = \sqrt{(x+5)^3},\ 0 \le x \le 8$.

(6) Compute the volume of the solid obtained by rotating the region $0 \le y \le xe^x$, $0 \le x \le 1$ around the OX axis.

(7) Compute the length of the curve $y = \log(x),\ 1 \le x \le \sqrt{3}$.

(8) Compute the volume of the solid obtained by rotating the region $\arctan(x) \le y \le \sqrt{\arctan^2(x) + 1 + \sin(x)}$, $0 \le x \le 2\pi$ around the OX axis.

(9) An orange has its ends cut off, so that the white flesh shows. The orange is then cut into slices of equal thickness. Show that each slice contains the same volume of the peel.

(10) Determine the convergence of the improper integrals ond compute those that are convergent:
(a) $\displaystyle\int_0^\infty \frac{dx}{x^2+1}$,
(b) $\displaystyle\int_0^4 \frac{dx}{\sqrt{x}}$,
(c) $\displaystyle\int_1^\infty \frac{dx}{\sqrt{x}}$,
(d) $\displaystyle\int_{-1}^1 \frac{x-1}{x^2-1}\,dx$,
(e) $\displaystyle\int_2^\infty \frac{dx}{x\log(x)}$,
(f) $\displaystyle\int_0^\infty \frac{dx}{e^{\sqrt[3]{x}}}$,
(g) $\displaystyle\int_0^\infty \cos(x)\,dx$,
(h) $\displaystyle\int_1^\infty x^{\frac{1}{x}}\,dx$,
(i) $\displaystyle\int_{-\infty}^\infty e^x\,dx$,
(j) $\displaystyle\int_0^1 e^{\frac{1}{x}}\,dx$,
(k) $\displaystyle\int_1^\infty \frac{e^{-\frac{1}{x}}}{x^3}\,dx$,
(l) $\displaystyle\int_2^\infty \frac{dx}{x\log^2(x)}$,
(m) $\displaystyle\int_0^\infty x^3\sin(x^4)\,dx$.