

CALCULUS
PROBLEMS LIST 5
29.10.2010

- (1) Compute the partial sums $s_n = \sum_{k=1}^n a_k$, and then find $\lim_{n \rightarrow \infty} s_n$:
- (a) $a_k = \frac{1}{5^k}$, (b) $a_k = \frac{2^k + 5^k}{10^k}$.
- (2) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$ is convergent, and its sum is less than 2.
- (3) Determine if the following series are convergent ($k!!$ denotes the product of all numbers not greater than k , of the same parity, and the function $\arctan$ will appear soon):
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| <p>(a) $\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$,</p> | <p>(b) $\sum_{n=2}^{\infty} \frac{1}{n^2 - 1}$,</p> |
| <p>(c) $\sum_{n=1}^{\infty} \frac{1+n}{n^2 + 1}$,</p> | <p>(d) $\sum_{n=1}^{\infty} \frac{2 \cdot 5 \cdot 8 \cdot \dots \cdot (3n-1)}{1 \cdot 5 \cdot 9 \cdot \dots \cdot (4n-3)}$,</p> |
| <p>(e) $\sum_{n=1}^{\infty} \frac{5n^2 - 1}{n^3 + 6n^2 + 8n + 47}$,</p> | <p>(f) $\sum_{n=1}^{\infty} \frac{1}{(2n-1) \cdot 2^{2n-1}}$,</p> |
| <p>(g) $\sum_{n=1}^{\infty} \frac{1}{3n-1}$,</p> | <p>(h) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2 + 2n}}$,</p> |
| <p>(i) $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+4)}$,</p> | <p>(j) $\sum_{n=1}^{\infty} \frac{1}{(2n+1)!}$,</p> |
| <p>(k) $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$,</p> | <p>(l) $\sum_{n=1}^{\infty} \frac{(2n-1)!!}{3^n n!}$,</p> |
| <p>(m) $\sum_{n=1}^{\infty} \left(\frac{n}{2n+1} \right)^n$,</p> | <p>(n) $\sum_{n=1}^{\infty} \frac{\left(\frac{n+1}{n} \right)^{n^3}}{3^n}$,</p> |
| <p>(o) $\sum_{n=2}^{\infty} \frac{1}{(n-1)\sqrt{n+1}}$,</p> | <p>(p) $\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n}}$,</p> |
| <p>(q) $\sum_{n=1}^{\infty} \frac{n^2}{n!}$,</p> | <p>(r) $\sum_{n=1}^{\infty} \frac{n}{2n-1}$,</p> |
| <p>(s) $\sum_{n=1}^{\infty} \frac{2^n}{n^4}$,</p> | <p>(t) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2 + n - n}}$,</p> |
| <p>(u) $\sum_{n=1}^{\infty} \frac{1000^n}{\sqrt[10]{n!}}$,</p> | <p>(v) $\sum_{n=1}^{\infty} \frac{\arctan n}{n^2 + \arctan n}$,</p> |
| <p>(w) $\sum_{n=1}^{\infty} \frac{3^n}{2^{2^n}}$,</p> | <p>(x) $\sum_{n=1}^{\infty} \frac{n^3 + \pi}{n^\pi + e}$.</p> |

(4) Which of the following series are convergent, and which are convergent absolutely:

$$(a) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1},$$

$$(b) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2 3^n},$$

$$(c) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)^3},$$

$$(d) \sum_{n=1}^{\infty} \frac{(-1)^{n+1} n + 1}{n},$$

$$(e) \sum_{n=1}^{\infty} \frac{1}{\sqrt{(n+4)(n+9)}},$$

$$(f) \sum_{n=1}^{\infty} \frac{(-1)^n \cdot 2^{10^n}}{3^{2^n}},$$

$$(g) \sum_{n=1}^{\infty} \frac{n! \cdot (-5)^n}{n^n \cdot 2^n},$$

$$(h) \sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^3}{2^n},$$

$$(i) 1 - 1 + 1 - \frac{1}{2} - \frac{1}{2} + 1 - \frac{1}{3} - \frac{1}{3} - \frac{1}{3} + \cdots + 1 - \overbrace{\frac{1}{k} - \frac{1}{k} - \cdots - \frac{1}{k}}^{k \text{ times}} + \cdots,$$

$$(j) 1 - 1 + \frac{1}{2} - \frac{1}{4} - \frac{1}{4} + \frac{1}{3} - \frac{1}{9} - \frac{1}{9} - \frac{1}{9} + \cdots + \frac{1}{k} - \overbrace{\frac{1}{k^2} - \frac{1}{k^2} - \cdots - \frac{1}{k^2}}^{k \text{ times}} + \cdots,$$

$$(k) \sum_{n=2}^{\infty} \frac{(-1)^n}{n - \sqrt{n}},$$

$$(l) \sum_{n=1}^{\infty} \frac{(-1)^{n+1} 2^{n^2}}{n!},$$

$$(m) \sum_{n=1}^{\infty} \frac{\sin 77n}{n^2},$$

$$(n) \sum_{n=1}^{\infty} \frac{2^n + 17}{3^n},$$

$$(o) \sum_{n=1}^{\infty} \frac{\sqrt{n! + 1}}{n!},$$

$$(p) \sum_{n=1}^{\infty} \frac{(-1)^{n^2}}{(n+3)^{1/4}},$$

$$(q) \sum_{n=1}^{\infty} \frac{n+2}{n(n+1)} (-1)^n,$$

$$(r) \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \left(1 + \frac{(-1)^n}{\sqrt{n}} \right),$$

$$(s) \sum_{n=1}^{\infty} \frac{2^n}{n\sqrt{4^n + 3^n}},$$

$$(t) \sum_{n=1}^{\infty} \frac{1}{n + 5\sqrt{n} + 27},$$

$$(u) \sum_{n=1}^{\infty} \frac{\binom{2n}{n}}{n!},$$

$$(v) \sum_{n=1}^{\infty} \frac{2^{n^2}}{4^{\binom{n}{2}}},$$

$$(w) \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/n}},$$

$$(x) \sum_{n=1}^{\infty} \frac{\left(\frac{n+1}{n}\right)^{n^2}}{2^n},$$

$$(y) \sum_{n=1}^{\infty} \frac{(-1)^n \left(\frac{n+1}{n}\right)^{n^2}}{3^n},$$

$$(z) \sum_{n=3}^{\infty} \frac{(\log n)^{\log n} (-1)^n}{n^{\log \log n}},$$

$$(\dot{z}) \sum_{n=1}^{\infty} \frac{(-1)^n}{\arctan n},$$

$$(\dot{z}) \sum_{n=1}^{\infty} (\sqrt{n+2} - \sqrt{n}) (-1)^n.$$