

MATHEMATICAL ANALYSIS

PROBLEMS LIST 8

26.11.10

- (1) Let $f(x) = \sqrt[3]{x^2}$. Using the definition compute the derivative $f'(8)$.
- (2) Let $f(x) = x^5$. Using the definition derive the formula for $f'(x)$.
- (3) Let $n \in \mathbf{N}$. Find constants a, b, c such that the function

$$f_n(x) = \begin{cases} |x| & : |x| \geq 1/n, \\ ax^2 + bx + c & : |x| < 1/n \end{cases}$$

is differentiable. Compute $f'_n(x)$. Sketch the graph of the function $f_n(x)$ and the graph of the derivative.

- (4) Compute the derivative of the following functions. Determine the set on which the derivative exists:

(a) $f(x) = 3x^2 - 5x + 1,$

(b) $f(x) = (\sqrt{x} + 1)\left(\frac{1}{\sqrt{x}} - 1\right),$

(c) $f(x) = \frac{1 - x^3}{1 + x^3},$

(d) $f(x) = (1 + \sqrt{x})(1 + x^{1/3})(1 + x^{1/4}),$

(e) $f(x) = (x^2 + 1)^4,$

(f) $f(x) = \frac{x + 1}{x - 1},$

(g) $f(x) = \frac{x}{x^2 + 1},$

(h) $f(x) = (1 + 2x)^{30},$

(i) $f(x) = \left(\frac{1}{1 + x^2}\right)^{1/3},$

(j) $f(x) = \frac{1}{\sqrt{1 - x^4 - x^8}},$

(k) $f(x) = 2^{x+3},$

(l) $f(x) = x10^x,$

(m) $f(x) = \frac{x}{e^x},$

(n) $f(x) = x^2(x + 1)e^x,$

(o) $f(x) = e^x \log x,$

(p) $f(x) = \frac{\log x}{e^x},$

(q) $f(x) = e^{x^2},$

(r) $f(x) = x^{10} \log x,$

(s) $f(x) = e^{e^x},$

(t) $f(x) = \log \log x,$

(u) $f(x) = \log_{10}(x - 1),$

(v) $f(x) = 10^{2x-3},$

(w) $f(x) = 2^{3^x},$

(x) $f(x) = \log_2 |\log_3 (\log_5 x)|,$

(y) $f(x) = e^{\sqrt{\log x}},$

(z) $f(x) = x^{x^2},$

(aa) $f(x) = x^{x^x},$

(ab) $f(x) = x^{\sqrt{x}},$

(ac) $f(x) = (\log x)^x,$

(ad) $f(x) = e^{-x^2} \log x,$

(ae) $f(x) = \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^{10},$

(af) $f(x) = x^5(x^6 - 8)^{1/3},$

(ag) $f(x) = e^{2x+3} \left(x^2 - x + \frac{1}{2}\right),$

(ah) $f(x) = \log \frac{1}{1 + x},$

(ai) $f(x) = \frac{e^{x^2}}{e^x + e^{-x}},$

(aj) $f(x) = |x|^3,$

(ak) $f(x) = \operatorname{sgn} x,$

(al) $f(x) = \begin{cases} 0 & \text{for } x < 0, \\ x^2 & \text{for } x \geq 0 \end{cases},$

- (am) $f(x) = e^{-|x|}$, (an) $f(x) = \sqrt{\sqrt{1+x^2}-1}$,
 (ao) $f(x) = \{x\}$, (ap) $f(x) = \begin{cases} x & \text{for } x < 0, \\ x^2 & \text{for } x \geq 0, \end{cases}$
 (aq) $f(x) = \operatorname{sgn}(x^5 - x^3)$, (ar) $f(x) = \frac{\pi^{10}}{\pi - e}$,
 (as) $f(x) = \begin{cases} e^x & \text{for } x < 0, \\ 1+x & \text{for } x \geq 0, \end{cases}$ (at) $f(x) = x^7 + e^2$,
 (au) $f(x) = (x+e)^{20}$, (av) $f(x) = e^e$.
- (5) A container is being designed in a shape of a drum with the top open, such that the bottom and the sides are made of the same material. The container is supposed to have a volume of 257 liters. What should be the ratio of the diameter of the bottom to the height of the container, so that the amount of material used to make it is minimal?
- (6) Find the minimal and the maximal value of the function given by the following formula in the given interval:
- (a) $f(x) = x^2 + 2x + 21$, $[-2, 7]$, (b) $f(x) = |x^2 - 1| + 3x$, $[-2, 2]$,
 (c) $f(x) = |x+1| + x^2$, $[-10, 10]$, (d) $f(x) = |10x-1| + x^3$, $[0, 1]$,
 (e) $f(x) = \log(x) - \frac{x}{10}$, $[1, e^3]$, (f) $f(x) = |\sin(x)| + \frac{x}{2}$, $[0, 2\pi]$,
 (g) $f(x) = x^{1/x}$, $[2, 4]$, (h) $f(x) = 3\sin(x) + \sin(3x)$, $[0, 2\pi]$.
- (7) Compute the limits:
- (a) $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\sin(x)} \right)$, (b) $\lim_{x \rightarrow \infty} x^{1/x}$,
 (c) $\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{\sin(x)}$, (d) $\lim_{x \rightarrow 0} \frac{2\cos(x) + x^2 - 2}{x\sin(x) - x^2}$,
 (e) $\lim_{x \rightarrow \infty} xe^{-x}$, (f) $\lim_{x \rightarrow \infty} \frac{\log(x)}{e^{e^x}x}$,
 (g) $\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$, (h) $\lim_{x \rightarrow 0} \frac{e^{e^x} - e}{x}$,
 (i) $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$, (j) $\lim_{x \rightarrow 1} \frac{\log(x)}{x-1}$,
 (k) $\lim_{x \rightarrow 1} \frac{\log(x) - x + 1}{(x-1)^2}$, (l) $\lim_{x \rightarrow e} \frac{\log \log(x)}{x-e}$,
 (m) $\lim_{x \rightarrow \infty} \frac{x^4}{e^x}$, (n) $\lim_{x \rightarrow 2} \frac{x^x - 4}{x-2}$.