

MATHEMATICAL ANALYSIS

PROBLEMS LIST 10

10.12.10

- (1) Find the radius of convergence of the Maclaurin series of functions:

$$(a) f(x) = \sqrt{x+2}, \quad (b) f(x) = \frac{1}{x+3}, \quad (c) f(x) = \log(x+e).$$

- (2) Find the inflection points and intervals of convexity of functions given by the formulas:

$$\begin{array}{lll} (a) & x^3 + 2x^2 + 3x + 4, & (b) \quad x^8 - x^2 + 7x - 15, \quad (c) \quad e^{-x^2}, \\ (d) & \sin^4(x), & (e) \quad \sqrt{x} - \log(x), \quad (f) \quad x^4 + \sqrt[4]{x}. \end{array}$$

- (3) Find the point of intersection of the tangent at the point $(2, 4)$ to the graph of the function $f(x) = x^2$ with the OY axis.

- (4) Find the point of intersection of the tangent at the point $(0, 1)$ to the graph of the function $f(x) = e^x$ with the OX axis.

- (5) Find the point of intersection of the tangents to the graph of $f(x) = x^3$ respectively at points $(-1, -1)$ and $(2, 8)$.

- (6) Compute $\int f(x) dx$ for $f(x)$ given by the formula:

$$\begin{array}{lll} (a) & 10^x, & (b) \quad \sqrt[n]{n}, \quad m, n \in \mathbf{N}, \quad (c) \quad a^x e^x, \quad a > 0, \\ (d) & 3, 4x^{-0,17}, & (e) \quad 1 - 2x, \quad (f) \quad \left(\frac{1-x}{x}\right)^2, \\ (g) & (\sqrt{x} + 1)(x - \sqrt{x} + 1), & (h) \quad \frac{\sqrt{x} - x^3 e^x + x^2}{x^3}, \quad (i) \quad (x+1)^{22}, \\ (j) & \frac{x^{100} - 1}{x - 1}, & (k) \quad \frac{x \sqrt[6]{x} + \sqrt[7]{x}}{x^2}, \quad (l) \quad \frac{x^3}{x+1}. \end{array}$$

- (7) Find a function F , such that $F''(x)$ is equal to:

$$(a) x^2 + 2x, \quad (b) \cos(x), \quad (c) e^{7x}.$$

- (8) Find a function F , such that:

$$\begin{array}{l} (a) F''(x) = x^2 + 1, \quad F'(0) = 2, \quad F(0) = 3, \\ (b) F''(x) = \frac{1}{x^3}, \quad F'(2) = 1, \quad F(3) = 5, \\ (c) F'''(x) = \sin(x), \quad F''(0) = F'(0) = F(0) = 0, \\ (d) F''(x) = \frac{1}{x^2}, \quad F'(1) = F'(-1) = 1, \quad F(1) = F(-1) = 3. \end{array}$$

- (9) Compute $\int f(x) dx$ for f given by the formula:

$$\begin{array}{lll} (a) & x \sin(2x), & (b) \quad x e^{-x}, \quad (c) \quad x^n \log(x), \quad n \in \mathbf{N}, \\ (d) & x^3 e^{5x}, & (e) \quad e^x \sin^2(x), \quad (f) \quad x 3^x, \\ (g) & x \sin(x) \cos(x), & (h) \quad e^{3x} \sin(2x), \quad (i) \quad \sqrt{e^x - 1}, \\ (j) & e^x \sin(e^x), & (k) \quad x e^{x^2}, \quad (l) \quad 1 \cdot \sin(\log(x)), \\ (m) & e^{-x^2} x, & (n) \quad \frac{\cos(\sqrt{x})}{\sqrt{x}}, \quad (o) \quad e^{\sqrt[3]{x}}, \end{array}$$

(p)	$\frac{1}{x \log(x) \log(\log(x))},$	(q)	$\cos(x) e^{\sin(x)},$	(r)	$6^{1-x},$
(s)	$\sin^5(x) \cos(x),$	(t)	$\frac{e^{2x}}{\sqrt[4]{e^x + 1}},$	(u)	$x e^{x^2} (x^2 + 1),$
(v)	$e^{5x} \sin(3x),$	(w)	$e^{5x} \cos(3x),$	(x)	$\sin(3x) \cdot \sin(5x),$
(y)	$\sin(15x) \cdot e^{-4x},$	(z)	$\frac{\arctan(x)}{x^2 + 1},$	(aa)	$\frac{\arctan^7(x) + 9 \arctan^5(x)}{x^2 + 1},$
(ab)	$\frac{x^3}{(x - 1)^{12}},$	(ac)	$\frac{\log^7(x) + \log^2(x)}{x},$	(ad)	$e^{-x^2} x^5,$
(ae)	$\sin(\sqrt{x}),$	(af)	$\frac{\sqrt{2 + \log(x)}}{x}.$		