CALCULUS

PROBLEMS LIST 5

25.10.2011

(1) Compute the partial sums $s_n = \sum_{k=1}^{n} a_k$, and then find $\lim_{n\to\infty} s_n$:

(a) $a_k = \frac{1}{5^k}$, (b) $a_k = \frac{2^k + 5^k}{10^k}$.

(2) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$ is convergent, and its sum is less than 2.

(3) Determine if the following series are convergent ($k!!$ denotes the product of all numbers not greater than k, of the same parity, and the function arctan will appear soon):

(a) $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$, (b) $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$,

(c) $\sum_{n=1}^{\infty} \frac{1+n}{n^2+1}$, (d) $\sum_{n=1}^{\infty} \frac{2\cdot 5\cdot 8\cdot\dots\cdot(3n-1)}{1\cdot 5\cdot 9\cdot\dots\cdot(4n-3)}$,

(e) $\sum_{n=1}^{\infty} \frac{5n^2-1}{n^3+6n^2+8n+47}$, (f) $\sum_{n=1}^{\infty} \frac{1}{(2n-1)\cdot 2^{2n-1}}$,

(g) $\sum_{n=1}^{\infty} \frac{1}{3n-1}$, (h) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+2n}}$,

(i) $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+4)}$, (j) $\sum_{n=1}^{\infty} \frac{1}{(2n+1)!}$,

(k) $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$, (l) $\sum_{n=1}^{\infty} \frac{(2n-1)!!}{3^n n!}$,

(m) $\sum_{n=1}^{\infty} \left(\frac{n}{2n+1}\right)^n$, (n) $\sum_{n=1}^{\infty} \frac{\left(\frac{n+1}{n}\right)^{n^3}}{3^n}$,

(o) $\sum_{n=2}^{\infty} \frac{1}{(n-1)\sqrt{n+1}}$, (p) $\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n}}$,

(q) $\sum_{n=1}^{\infty} \frac{n^2}{n!}$, (r) $\sum_{n=1}^{\infty} \frac{n}{2n-1}$,

(s) $\sum_{n=1}^{\infty} \frac{2^n}{n^4}$, (t) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+n}-n}$,

(u) $\sum_{n=1}^{\infty} \frac{1000^n}{\sqrt[10]{n!}}$, (v) $\sum_{n=1}^{\infty} \frac{\arctan n}{n^2+\arctan n}$,

(w) $\sum_{n=1}^{\infty} \frac{3^n}{2^{2^n}}$, (x) $\sum_{n=1}^{\infty} \frac{n^3+\pi}{n^\pi+e}$.

(4) Which of the following series are convergent, and which are convergent absolutely:

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$, (b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2 3^n}$,

(c) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)^3}$, (d) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n+1}{n}$,

(e) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{(n+4)(n+9)}}$, (f) $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot 2^{10^n}}{3^{2^n}}$,

(g) $\sum_{n=1}^{\infty} \frac{n! \cdot (-5)^n}{n^n \cdot 2^n}$, (h) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n^3}{2^n}$,

(i) $1 - 1 + 1 - \frac{1}{2} - \frac{1}{2} + 1 - \frac{1}{3} - \frac{1}{3} - \frac{1}{3} + \dots + 1 - \overbrace{\frac{1}{k} - \frac{1}{k} - \dots - \frac{1}{k}}^{k \text{ times}} + \dots$,

(j) $1 - 1 + \frac{1}{2} - \frac{1}{4} - \frac{1}{4} + \frac{1}{3} - \frac{1}{9} - \frac{1}{9} - \frac{1}{9} + \dots + \frac{1}{k} - \overbrace{\frac{1}{k^2} - \frac{1}{k^2} - \dots - \frac{1}{k^2}}^{k \text{ times}} + \dots$,

(k) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n-\sqrt{n}}$, (l) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}2^{n^2}}{n!}$,

(m) $\sum_{n=1}^{\infty} \frac{\sin 77n}{n^2}$, (n) $\sum_{n=1}^{\infty} \frac{2^n+17}{3^n}$,

(o) $\sum_{n=1}^{\infty} \frac{\sqrt{n!+1}}{n!}$, (p) $\sum_{n=1}^{\infty} \frac{(-1)^{n^2}}{(n+3)^{1/4}}$,

(q) $\sum_{n=1}^{\infty} \frac{n+2}{n(n+1)}(-1)^n$, (r) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}\left(1 + \frac{(-1)^n}{\sqrt{n}}\right)$,

(s) $\sum_{n=1}^{\infty} \frac{2^n}{n\sqrt{4^n+3^n}}$, (t) $\sum_{n=1}^{\infty} \frac{1}{n+5\sqrt{n}+27}$,

(u) $\sum_{n=1}^{\infty} \frac{\binom{2n}{n}}{n!}$, (v) $\sum_{n=1}^{\infty} \frac{2^{n^2}}{4^{\binom{n}{2}}}$,

(w) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/n}}$, (x) $\sum_{n=1}^{\infty} \frac{\left(\frac{n+1}{n}\right)^{n^2}}{2^n}$,

(y) $\sum_{n=1}^{\infty} \frac{(-1)^n\left(\frac{n+1}{n}\right)^{n^2}}{3^n}$, (z) $\sum_{n=3}^{\infty} \frac{(\log n)^{\log n}(-1)^n}{n^{\log\log n}}$,

(ż) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\arctan n}$, (ź) $\sum_{n=1}^{\infty} \left(\sqrt{n+2} - \sqrt{n}\right)(-1)^n$.