CALCULUS
PROBLEMS LIST 9
29.11.2011

(1) Let
$$f(x)=\begin{cases}\dfrac{e^{x^2}-1}{\cos x-1} & : \quad x\neq 2k\pi,\ k\in\mathbf{Z},\\ A & : \quad x=0.\end{cases}$$
For which A does the derivative $f'(0)$ exist and how much is it?

(2) Let
$$f(x)=\begin{cases}\dfrac{\sin x-1}{\cos^2 x} & : \quad x\notin\{k\pi+\frac{\pi}{2};k\in\mathbf{Z}\},\\ A_k & : \quad x=k\pi+\frac{\pi}{2},\ k\in\mathbf{Z}.\end{cases}$$
For which A_k ($k\in\mathbf{Z}$) do the derivatives $f'(k\pi+\frac{\pi}{2})$ exist and how much are they?

(3) Let
$$f(x)=\begin{cases}\dfrac{x(x-1)(x-2)(x-3)}{\sin(\pi x)} & : \quad x\notin\mathbf{Z},\\ x^2-2x & : \quad x=\mathbf{Z}.\end{cases}$$
Compute $f'(x)$ for those $x\in\mathbf{Z}$, for which it exists.

(4) Let
$$f(x)=\begin{cases}\dfrac{e^{7x}-1}{x} & : \quad x\neq 0,\\ 7 & : \quad x=0.\end{cases}$$
Compute $f'(0)$.

(5) Let
$$f(x)=\begin{cases}\dfrac{\cos(\pi x)+1}{\sin(\pi x)} & : \quad x\notin\mathbf{Z},\\ x^3-x & : \quad x\in\mathbf{Z}.\end{cases}$$
Compute $f'(x)$ for those $x\in\mathbf{Z}$, for which it exists.

(6) Let
$$f(x)=\begin{cases}\dfrac{e^{3x}-3e^{x}+2}{x^2} & : \quad \neq 0,\\ A & : \quad x=0.\end{cases}$$
For which A does the derivative $f'(0)$ exist and how much is it?

(7) Compute the derivative of order 3 of the function f, given by the formula:

(a) $(x+1)^6$, (b) x^6-4x^3+4, (c) $\dfrac{1}{1-x}$,
(d) $x^3\log x$, (e) e^{2x-1}; (f) $(x^2+1)^3$,
(g) e^{x^2}, (h) $\log(x^2)$, (i) $(x-7)^{50}$.

(8) Derive the formula for the derivative of order n for the function f given by the formula:

(a)	$\log(x^{10})$,	(b)	$x\log(x)$,	(c)	$\sqrt{x}$,
(d)	$\sin^2(x)$,	(e)	$\dfrac{1-x}{1+x}$,	(f)	xe^x,
(g)	$\sin(5x)$,	(h)	x^7,	(i)	e^{4x},
(j)	$x+\dfrac{1}{x}$,	(k)	x^2e^{-x}.		

(9) Prove that

$$(f\cdot g)^{(n)}(x)=\sum_{k=0}^{n}\binom{n}{k}f^{(k)}(x)g^{(n-k)}(x).$$

(10) Compute approximate values of the following expressions, using the three initial terms (zeroth, first and second) of appropriately set up Taylor series. Estimate the error of the approximation using the Taylor's formula:

(a)	$\sqrt{24}$,	(b)	$\sqrt[3]{126}$,	(c)	$\sqrt[7]{126}$,
(d)	$\sin(\frac{1}{10})$,	(e)	$\arctan(\frac{1}{10})$,	(f)	$\sqrt{50}$.