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A Decomposition Theorem for Compact-Valued Henstock Integral

By

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Abstract. We prove that if X is a separable Banach space, then a measurable multifunction $\Gamma : [0,1] \to ck(X)$ is Henstock integrable if and only if Γ can be represented as $\Gamma = G + f$, where $G : [0,1] \to ck(X)$ is McShane integrable and f is a Henstock integrable selection of Γ.

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1. Introduction

There is a great deal of literature on Bochner and Pettis integration of Banach space valued multifunctions (see El Amri and Hess [4] or Hess and Ziat [8] for further references) of several types that have shown to be a useful tool in many branches of mathematics such as mathematical economy, control theory and optimization.

Quite recently Ziat [13, 14] and El Amri and Hess [4] presented several criteria for a multifunction having as its values convex weakly compact subsets of a Banach space, to be Pettis integrable. In [3] we studied the obvious generalization of the Pettis integral of a multifunction obtained by replacing the Lebesgue integrability of the support functions by the Kurzweil–Henstock integrability (we call such an integral Kurzweil–Henstock–Pettis). We proved there a surprising and unexpected characterization of the new integral in terms of the Pettis integral: the Kurzweil–Henstock–Pettis integral is in some way a translation of the Pettis one.

In this paper we deal with the Henstock and McShane integrals for multifunctions. Such integrals are generalizations, by means the notion of Hausdorff distance, of the definitions of Henstock and McShane integrals for vector valued functions. We present (see Theorem 2) a characterization of Henstock integrable multifunctions with convex compact values, similar to that in [3]: each Henstock integrable multifunction is the sum of one of its Henstock integrable selection and of a McShane integrable multifunction. The proof is based on the Rådström embedding of the space of all compact convex subsets of the target separable Banach space into a suitable separable Banach space.

We show also (see Theorem 2) a criterion of Henstock integrability similar to that given in case of Pettis integrability (see [4]): a multifunction is Henstock integrable if and only if all its selections are Henstock integrable.

We prove also that if the multifunctions are compact convex valued and the target Banach space is separable, then the Pettis and the McShane integrals coincide (see Proposition 2), as in case of functions taking their values in a separable Banach space (cf [6]).

2. Basic Facts

Let [0, 1] be the unit interval of the real line equipped with the usual topology and the Lebesgue measure. $\mathscr{L}$ denotes the family of all Lebesgue measurable subsets of [0, 1] and if $E \in \mathscr{L}$, then $|E|$ denotes its Lebesgue measure. A *partition P of* [0, 1] is a collection $\{(I_1, t_1), \ldots, (I_p, t_p)\}$, where $I_1, \ldots, I_p$ are nonoverlapping subintervals of [0, 1], t_i is a point of [0, 1], $i = 1, \ldots, p$ and $\bigcup_{i=1}^{p} I_i = [0, 1]$. If t_i is a point of I_i, $i = 1, \ldots, p$, we say that P is a *Perron partition of* [0, 1]. A *gauge* on [0, 1] is a positive function on [0, 1]. For a given gauge δ on [0, 1], we say that a partition $\{(I_1, t_1), \ldots, (I_p, t_p)\}$ is *δ-fine* if $I_i \subset (t_i - \delta(x_i), t_i + \delta(x_i))$, $i = 1, \ldots, p$.

Definition 1 (see [1], [5] and [6]). Let Y be a Banach space. A function $g : [0, 1] \to Y$ is said to be *Henstock* (resp. *McShane*) *integrable* on [0, 1] if there exists $w \in Y$ with the following property: for every $\epsilon > 0$ there exists a gauge δ on [0, 1] such that

$$\left\| \sum_{i=1}^{p} g(t_i)|I_i| - w \right\| < \varepsilon,$$

for each δ-fine Perron partition (resp. partition) $\{(I_1, t_1), \ldots, (I_p, t_p)\}$ of [0, 1]. We set $w =: (H) \int_0^1 g dt$ (resp. $w =: (Mc) \int_0^1 g dt$).

We denote the set of all Henstock (resp. McShane) integrable functions on [0, 1], taking their values in Y, by $\mathscr{H}([0, 1], Y)$ (resp. $\mathscr{M}c([0, 1], Y)$). In case when Y is the real line, g is called *Kurzweil–Henstock integrable*, or simply *KH-integrable*, (resp. integrable). Moreover the space of all KH-integrable functions (resp. integrable functions) is denoted by $\mathscr{K}\mathscr{H}[0, 1]$ (resp. $\mathscr{M}c[0, 1]$).

Definition 2. Let $\{g_\alpha\}$ be a family of real valued functions in $\mathscr{K}\mathscr{H}[0, 1]$ (resp. $\mathscr{M}c[0, 1]$). We say that $\{g_\alpha\}$ is Henstock (resp. McShane) equiintegrable on [0, 1] whenever for every $\varepsilon > 0$ there is a gauge δ such that

$$\sup_{\alpha} \left\{ \left| \sum_{i=1}^{p} g_\alpha(t_i)|I_i| - (HK) \int_0^1 g_\alpha dt \right| \right\} < \varepsilon,$$

for each δ-fine Perron partition (resp. partition) $\{(I_1, t_1), \ldots, (I_p, t_p)\}$ of [0, 1].

Throughout this paper X is a separable Banach space with its dual X^*. The closed unit ball of X^* is denoted by $B(X^*)$. $ck(X)$ denotes the family of all nonempty convex compact subsets of X. For every $C \in ck(X)$ the *support function of* C is denoted by $s(\cdot, C)$ and defined on X^* by $s(x^*, C) = \sup\{\langle x^*, x\rangle : x \in C\}$, for each $x^* \in X^*$. A map $\Gamma : [0,1] \to 2^X \setminus \{\emptyset\}$ (=closed non-empty subsets of X) is called a *multifunction*. A multifunction Γ is said to be *measurable* if for each open subset O of X, the set $\{t \in [0,1] : \Gamma(t) \cap O \neq \emptyset\}$ is a measurable set. Γ is said to be *scalarly measurable* if for every $x^* \in X^*$, the map $s(x^*, \Gamma(\cdot))$ is measurable. It is however well known that in case of $ck(X)$-valued multifunctions the scalar measurability yields the measurability. Γ is said to be *scalarly Kurzweil–Henstock* (resp. *scalarly*) *integrable* if, for every $x^* \in X^*$, the function $s(x^*, \Gamma(\cdot))$ is *Kurzweil–Henstock integrable* (resp. *integrable*). A function $f : [0,1] \to X$ is called a *selection of* Γ if, for every $t \in [0,1]$, one has $f(t) \in \Gamma(t)$. A selection f is said to be *measurable* if the function f is strongly measurable. Notice that due to the separability of X, Borel measurability, weak measurability and strong measurability of X-valued functions are equivalent. It is a consequence of [10] that each measurable multifunction $\Gamma : [0,1] \to ck(X)$ has a measurable selection. By $\mathcal{S}_H(\Gamma)$ (resp. $\mathcal{S}_{Mc}(\Gamma)$) we denote the family of all measurable selections of Γ that are Henstock (resp. McShane) integrable.

Let A and B be nonempty subsets of X. The *excess* of A over B is defined as $e(A,B) = \sup\{d(x,B) : x \in A\}$. The *Hausdorff distance of* A and B is defined by $d_H(A,B) := \max\{e(A,B), e(B,A)\}$. $ck(X)$ with the Hausdorff distance is a complete metric space. We consider on $ck(X)$ the Minkowski addition ($A + B := \{a + b : a \in A,\ b \in B\}$) and the standard multiplication by scalars.

Definition 3. A multifunction $\Gamma : [0,1] \to ck(X)$ is Henstock (resp. McShane) integrable in $ck(X)$, if there exists a nonempty set $W \in ck(X)$ with the following property: for every $\varepsilon > 0$ there exists a gauge δ on [0, 1] such that for each δ-fine Perron partition (resp. partition) $\{(I_1, t_1), \dots, (I_p, t_p)\}$ of [0, 1], we have

$$d_H\left(W, \sum_i \Gamma(t_i)|I_i|\right) < \varepsilon.$$

We note that when a multifunction is a function $f : [0,1] \to X$, then the set W is reduced to a vector of X and the above definitions coincide with those of Henstock and McShane integrability for vector valued functions.

It can be easily seen that if Y is a Banach space and $i : (ck(X), d_H) \to Y$ is a linear isometry embedding the space $(ck(X), d_H)$ into Y, then the multifunction $\Gamma : [0,1] \to ck(X)$ is Henstock (resp. McShane) integrable if and only if $i(\Gamma)$ is Henstock (resp. McShane) integrable as an Y-valued function.

Definition 4. A multifunction $\Gamma : [0,1] \to ck(X)$ is Kurzweil–Henstock–Pettis integrable (see [3]) in $ck(X)$ (resp. Pettis integrable (see [4]) in $ck(X)$) if Γ is scalarly Kurzweil–Henstock (resp. scalarly) integrable on [0, 1] and for each subinterval $[a,b]\subset[0,1]$ (resp. $\emptyset\neq A\in\mathscr{L}$), there exists a set $W_{[a,b]}\in ck(X)$ (resp. $W_A\in ck(X)$) such that for each $x^*\in X^*$, we have

$$\begin{aligned} s(x^*, W_{[a,b]}) &= (HK)\int_a^b s(x^*,\Gamma(t))\,dt \\ (\text{resp. } s(x^*, W_A) &= \int_A s(x^*,\Gamma(t))\,dt\,), \end{aligned} \tag{1}$$

where $\int_A$ stands for the Lebesgue integral. $\mathscr{S}_{\mathrm{KHP}}(\Gamma)$ denotes the family of all measurable selections of Γ that are Kurzweil–Henstock–Pettis integrable.

According to Hörmander's equality (cf [9], p. 9) we have the equality

$$d_H\left(K,\sum_{i=1}^p \Gamma(t_i)|I_i|\right) = \sup_{\|x^*\|\leqslant 1}\left|s(x^*,K)-\sum_{i=1}^p s(x^*,\Gamma(t_i))|I_i|\right|, \tag{2}$$

for each δ-fine Perron partition (resp. partition) $\{(I_1,t_1),\dots,(I_p,t_p)\}$ of [0, 1] and set $K\in ck(X)$. It follows that if Γ is Henstock (resp. McShane) integrable, then it is also Kurzweil–Henstock–Pettis (resp. Pettis) integrable. We note that in case the multifunction is a function $f:[0,1]\to X$ the sets $W_{[a,b]}$ (resp. W_A) are reduced to vectors in X.

Proposition 1. *If* $\Gamma:[0,1]\to ck(X)$ *is a scalarly KH-integrable* (*resp. scalarly integrable*) *multifunction*, *then the following conditions are equivalent*:

(i) Γ *is Henstock* (*resp. McShane*) *integrable in* $ck(X)$;
(ii) *the collection* $\{s(x^*,\Gamma(\cdot)):\|x^*\|\leqslant 1\}$ *is Henstock* (*resp. McShane*) *equi-integrable*;
(iii) *each countable subset of the collection* $\{s(x^*,\Gamma(\cdot)):\|x^*\|\leqslant 1\}$ *is Henstock* (*resp. McShane*) *equiintegrable*.

Proof. (i) $\Rightarrow$ (ii) It is enough to use Hörmander's equality (2).
(ii) $\Rightarrow$ (iii) is obvious.
(iii) $\Rightarrow$ (i) We are going to show first that there exists $K\in ck(X)$ such that $(H)\int_0^1 s(x^*,\Gamma)=s(x^*,K)$ (resp. $\int_0^1 s(x^*,\Gamma)=s(x^*,K)$) for all functionals x^*. We shall prove first that the sublinear function $a:X^*\to(-\infty,+\infty)$ given by $a(x^*):=(KH)\int_0^1 s(x^*,\Gamma(t))\,dt$ (resp. $a(x^*):=\int_0^1 s(x^*,\Gamma(t))\,dt$) is w^*-lower semi-continuous, i.e. that for each real α the set $Q(\alpha):=\{x^*\in X^*: a(x^*)\leqslant\alpha\}$ is w^*-closed. According to the Banach-Dieudonné theorem it suffices to show that $Q(\alpha)\cap B(X^*)$ is w^*-closed. So consider a sequence of functionals $x_n^*\in Q(\alpha)\cap B(X^*)$. Assuming that $x_n^*\to x_0^*$ in $\sigma(X^*,X)$ and applying the w^*-continuity of all $s(\cdot,\Gamma(t))$, we get the pointwise convergence of $s(x_n^*,\Gamma(t))$ to $s(x_0^*,\Gamma(t))$. Since the sequence is Henstock (resp. McShane) equiintegrable, we have

$$a(x_0^*) = (KH)\int_0^1 s(x_0^*,\Gamma)\,dt = \lim_n\,(KH)\int_0^1 s(x_n^*,\Gamma)\,dt = \lim_n a(x_n^*)\leqslant\alpha,$$

(resp.

$$a(x_0^*) = \int_0^1 s(x_0^*, \Gamma)\,dt = \lim_n \int_0^1 s(x_n^*, \Gamma)\,dt = \lim_n a(x_n^*) \leqslant \alpha).$$

Consequently, the function a is w^*-continuous (hence it is also w^*-lower semicontinuous) and so, according to a result of Hörmander, there exists a closed convex set $K \subset X$ such that $a(x^*) = s(x^*, K)$. The weak*-continuity of a yields the norm compactness of K. Moreover, if $\{x_n^* : n \in \mathbf{N}\}$ is a w^*-dense subset of $B(X^*)$, then according to the assumption, the family $\{s(x_n^*, \Gamma(\cdot)\}$ is Henstock (resp. McShane) equiintegrable and

$$\sup_{\|x^*\| \leqslant 1} \left| s(x^*, K) - \sum_{i \leqslant p} s(x^*, \Gamma(t_i))|I_i| \right| = \sup_n \left| s(x_n^*, K) - \sum_{i \leqslant p} s(x_n^*, \Gamma(t_i))|I_i| \right|.$$

Indeed, if $x^* \in B(X^*)$ is arbitrary, then there is $x_{n_k}^* \to x^*$ in the w^*-topology. If $\{(I_1, t_1), \ldots, (I_p, t_p)\}$ is a Perron partition (resp. partition) of [0, 1], then $\lim_k s(x_{n_k}^*, K) = s(x^*, K)$ and $\lim_k s(x_{n_k}^*, \Gamma(t_i)) = s(x^*, \Gamma(t_i))$ for all $i \leqslant p$. It follows that

$$\lim_k \left| s(x_{n_k}^*, K) - \sum_{i=1}^{p} s(x_{n_k}^*, \Gamma(t_i))|I_i| \right| = \left| s(x^*, K) - \sum_{i=1}^{p} s(x^*, \Gamma(t_i))|I_i| \right|$$

and so the required equality holds. Applying once again (2), we get the Henstock (resp. McShane) integrability of Γ in $ck(X)$. □

3. A Decomposition Theorem

Let $\widehat{X}$ be the closed subspace of $C(B(X^*), \sigma(X^*, X))$ generated by $\{s(\cdot, C) : C \in ck(X)\}$. Then $\widehat{X}$ is a separable Banach space. Let $R : ck(X) \to \widehat{X}$ be the Rådström embedding (see [12]), defined by $R(C) = s(\cdot, C)$, for any $C \in ck(X)$.

Lemma 1. *If $\Gamma : [0,1] \to ck(X)$ is a Pettis integrable multifunction in $ck(X)$, then $R(\Gamma)$ is Pettis integrable in $\widehat{X}$ and $\int_A R(G(t))\,dt = R(\int_A G(t)\,dt)$, for every $A \in \mathscr{L}$.*

Proof. As Γ is Pettis integrable, for each $A \in \mathscr{L}$ there exists a nonempty set $W_A \in ck(X)$ such that for each $x^* \in X^*$, we have

$$s(x^*, W_A) = \int_A s(x^*, \Gamma(t))\,dt.$$

Let now $y^* \in \widehat{X}^*$. Then its extension to $C(B(X^*))$ (denote it also by y^*) is a Radon measure on $(B(X^*), \sigma(X^*, X))$. If s is considered only for $x^* \in B(X^*)$, then s is a Caratheodory function i.e. $s(x^*, \cdot)$ is Lebesgue measurable and $s(\cdot, \Gamma(t))$ is weak*-continuous. Consequently s is measurable with respect to the product σ-algebra $\mathscr{B}(B(X^*), \sigma(X^*, X)) \otimes \mathscr{L}$, where $\mathscr{B}(B(X^*), \sigma(X^*, X))$ is the σ-algebra of Borel subsets of $B(X^*)$ in the weak*-topology (cf [9], Prop. 2.1.6). From the Riesz representation theorem y^* is a bounded measure on $\mathscr{B}(B(X^*), \sigma(X^*, X))$. As $W_A \in ck(X)$, we have

$$\sup_{x^* \in B(X^*)} |s(x^*, W_A)| = \sup_{x^* \in B(X^*)} \sup_{x \in W_A} |x^*(x)| < \infty.$$

Then, due to the Nikodym boundedness theorem

$$\begin{aligned} \int_0^1 |s(x^*, \Gamma(t))|\, dt &\leqslant \sup_{E \in \mathscr{L}} \left| \int_E s(x^*, \Gamma(t)\, dt \right| + \sup_{E \in \mathscr{L}} \left| \int_E s(-x^*, \Gamma(t)\, dt \right| \\ &= \sup_{E \in \mathscr{L}} |s(x^*, W_E)| + \sup_{E \in \mathscr{L}} |s(-x^*, W_E)| < M, \end{aligned}$$

for a suitable constant M.

Consequently, $\int_{B(X^*)} \int_0^1 |s(x^*, \Gamma(t))|\, dt\, dy^* < \infty$ and we may apply the Fubini-Tonelli theorem. If $A \in \mathscr{L}$, then

$$\begin{aligned} \int_A y^*(R(\Gamma(t))dt &= \int_A \left(\int_{B(X^*)} (R(\Gamma(t))dy^* \right) dt \\ &= \int_A \left(\int_{B(X^*)} s(x^*, \Gamma(t))dy^* \right) dt \\ &= \int_{B(X^*)} \left(\int_A s(x^*, \Gamma(t))dt \right) dy^* \\ &= \int_{B(X^*)} s(x^*, W_A) dy^* = y^*(R(W_A)). \end{aligned}$$

Thus, $R(G)$ is Pettis integrable, and $\int_A R(G(t))\, dt) = R(\int_A G(t)\, dt)$. □

Proposition 2. *A multifunction $\Gamma : [0,1] \to ck(X)$ is Pettis integrable in $ck(X)$ if and only if it is McShane-integrable in $ck(X)$.*

Proof. If Γ is Pettis integrable, then, according to Lemma 1 the function $R(\Gamma)$ is Pettis integrable in $\widehat{X}$. As $\widehat{X}$ is separable, it follows from [6] or [7] that $R(\Gamma)$ is McShane integrable in $\widehat{X}$. Consequently, Γ is McShane integrable in $ck(X)$. The reverse implication follows exactly in the same way, as in separable Banach spaces Pettis and McShane integrability are equivalent (see [6, 7]). □

Theorem 2. *Let $\Gamma : [0,1] \to ck(X)$ be a scalarly Kurzweil–Henstock integrable multifunction. Then the following conditions are equivalent*:

(i) *Γ is Henstock integrable in $ck(X)$;*

(ii) *$\mathscr{S}_H(\Gamma) \neq \emptyset$ and for every $f \in \mathscr{S}_H(\Gamma)$ the multifunction $G : [0,1] \to ck(X)$ defined by $\Gamma(t) = G(t) + f(t)$ is McShane integrable in $ck(X)$;*

(iii) *there exists $f \in \mathscr{S}_H(\Gamma)$ such that the multifunction $G : [0,1] \to ck(X)$ defined by $\Gamma(t) = G(t) + f(t)$ is McShane integrable in $ck(X)$;*

(iv) *every measurable selection of Γ is Henstock integrable.*

If $c_0 \nsubseteq X$, then the above conditions are equivalent also to:

(v) $\mathscr{S}_H(\Gamma) \neq \emptyset$.

Proof. (i) $\Rightarrow$ $\{$(ii), (iii), (iv)$\}$ Each Henstock integrable multifunction is also Kurzweil–Henstock–Pettis integrable and so according to Theorem 1 from [3], each measurable selection f of Γ is a KHP-integrable function. So let $f \in \mathscr{S}_{\mathrm{KHP}}(\Gamma)$ be arbitrary and the multifunction $G : [0,1] \to ck(X)$ be defined by $\Gamma(t) = G(t) + f(t)$. According to [3] Theorem 1, G is Pettis integrable in $ck(X)$ and so it is a consequence of Proposition 2 that G itself is McShane integrable in $ck(X)$. Then G is also Henstock integrable $ck(X)$. Therefore, by Proposition 1, both families of functions $\{s(x^*, \Gamma(\cdot)) : \|x^*\| \leqslant 1\}$ and $\{s(x^*, G(\cdot)) : \|x^*\| \leqslant 1\}$ are Henstock equiintegrable. Since for each $x^* \in X^*$ we have

$$s(x^*, \Gamma(\cdot)) = s(x^*, G(\cdot)) + \langle x^*, f(\cdot)\rangle, \tag{3}$$

also the family $\{\langle x^*, f(\cdot)\rangle : \|x^*\| \leqslant 1\}$ is Henstock equiintegrable. Applying once again Proposition 1 we obtain the Henstock integrability of f. Thus, (ii) and (iv) are satisfied. The implication (ii) $\Rightarrow$ (iii) is obvious.

(iv) $\Rightarrow$ (i) Let f be a Henstock integrable selection of Γ. So $f \in \mathscr{S}_{\mathrm{KHP}}(\Gamma)$ and, according to Theorem 1 from [3], Γ is Kurzweil–Henstock–Pettis integrable in $ck(X)$ and the multifunction G defined by $G(t) = \Gamma(t) - f(t)$ is Pettis integrable in $ck(X)$. As in the first part of the proof we get the McShane integrability of G in $ck(X)$ and consequently, the Henstock integrability of G in $ck(X)$. Therefore, by Proposition 1, the family $\{s(x^*, G(\cdot)) : \|x^*\| \leqslant 1\}$ is Henstock equiintegrable. The same proposition yields the Henstock equiintegrability of the collection $\{\langle x^*, f(\cdot)\rangle : \|x^*\| \leqslant 1\}$. Then, the equality (3) yields the Henstock equiintegrability of $\{s(x^*, \Gamma(\cdot)) : \|x^*\| \leqslant 1\}$. Applying once again Proposition 1 one obtains the Henstock integrability of Γ.

In a similar way (iii) $\Rightarrow$ (i) can be proved.

Assume now that $c_0 \nsubseteq X$ and (v) is satisfied. Let $f \in \mathscr{S}_H(\Gamma)$ and let G be defined as before. Then, by (3), taking into account non-negativity of $s(x^*, G(\cdot))$, we obtain the integrability of $s(x^*, G(\cdot))$, for each $x^* \in X^*$. Since for each measurable selection g of G, $|\langle x^*, g(\cdot)\rangle| \leqslant s(x^*, G(\cdot)) + s(-x^*, G(\cdot))$, each measurable selection of G is scalarly integrable, hence Pettis integrable (because $c_0 \nsubseteq X$). Applying Theorem 4.4 of [14] we obtain the Pettis, and then (see Proposition 2) the McShane integrability of G. This yields the Henstock integrability of Γ in $ck(X)$. □

If c_0 can be isomorphically embedded into X, then the condition (v) of Theorem 2 is not sufficient for the Henstock integrability of Γ.

Example 1. Let $f : [0,1] \to c_0$ be defined by

$$f(t) = \left(\chi_{(0,1]}(t), 2\chi_{(0,\frac{1}{2}]}(t), \ldots, n\chi_{(0,\frac{1}{n}]}(t), \ldots\right)$$

(cf [2], p. 53). Then f is scalarly integrable but not Pettis integrable because Dunford – $\int_0^1 f(t)\,dt \in l_\infty \backslash c_0$. The last relation shows also that f is neither KHP-integrable nor Henstock integrable. If $\Gamma(t) := \mathrm{conv}\{0, f(t)\}$, then Γ is scalarly integrable (hence also scalarly KH-integrable) and $\mathscr{S}_H(\Gamma) \neq \emptyset$. f is a measurable selection of Γ that is not Henstock integrable and so Γ is not Henstock integrable in $ck(X)$ (according to Theorem 2).

References

[1] Cao SS (1992) The Henstock integral for Banach-valued functions. Southeast Asian Bull Math **16**: 35–40
[2] Diestel J, Uhl JJ (1977) Vector Measures. Math Surveys **15**. Providence, RI: Amer Math Soc
[3] Di Piazza L, Musiał K (2005) Set-Valued Kurzweil–Henstock–Pettis Integral. Set-Valued Anal **13**: 167–179
[4] El Amri K, Hess C (2000) On the Pettis integral of closed valued multifunctions. Set-Valued Anal **8**: 329–360
[5] Fremlin DH (1994) The Henstock and McShane integrals of vector-valued functions. Illinois J Math **38**: 471–479
[6] Fremlin DH (1995) The generalized McShane integral. Illinois J Math **39**: 39–67
[7] Gordon R (1990) The McShane integral of Banach-valued functions. Illinois J Math **34**: 557–567
[8] Hess C, Ziat H (2002) Théorèm de Komlós pour des multifonctions intégrables au sense de Pettis et applications. Ann Sci Math Quèbec **26**: 181–198
[9] Hu S, Papageorgiou NS (1997) Handbook of Multivalued Analysis I. Amsterdam Kluwer
[10] Kuratowski K, Ryll-Nardzewski C (1965) A general theorem on selections. Bull Acad Polon Sci Sèr Sci Math Astronom Phys **13**: 397–403
[11] Musial K (1991) Topics in the theory of Pettis integration. Rend Istit Mat Univ Trieste **23**: 177–262
[12] Rådström H (1952) An embedding theorem for spaces of convex sets. Proc Amer Math Soc **3**: 165–169
[13] Ziat H (1997) Convergence theorems for Pettis integrable multifunctions. Bull Polish Acad Sci Math **45**: 123–137
[14] Ziat H (2000) On a characterization of Pettis integrable multifunctions. Bull Polish Acad Sci Math **44**: 227–230

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