DST 2 2024

Zad. 1 Show that $(2^\omega)^\omega$ is homeomorphic to 2^ω.

Zad. 2 Consider the space of continuous functions $f\colon 2^\omega \to 2^\omega$ (with the supremum metric). Is it homeomorphic to 2^ω?

Zad. 3 Let X be a Polish space and let F be an F_σ subset of X. Show that for every $\varepsilon > 0$ there is a sequence (F_n) of pairwise disjoint F_σ sets such that

a) $diam(F_n) < \varepsilon$ for each n,

b) $\overline{F_n} \subseteq F$ for each n,

c) $\bigcup_n F_n = F$.

Zad. 4 Use the above to show that in every non-empty Polish space we can perform a Lusin scheme $(F_s)_{s\in\omega^{<\omega}}$ such that

- $F_\emptyset = X$,
- F_s is F_σ for each $s \in \omega^{<\omega}$,
- $F_s = \bigcup_n F_{s^\frown n}$ for each $s \in \omega^{<\omega}$.

Zad. 5 Use the above to show that for each Polish space X there is a closed set $F \subseteq \omega^\omega$ and a continuous bijection $f\colon F \to X$.

Zad. 6 Use the above to show that every Polish space is a continuous image of ω^ω.

Zad. 7 (Brouwer theorem) Show that 2^ω is the unique (modulo homeomorphism) nonempty, perfect, compact metrizable, zero-dimensional space. (Recall that a space is zerodimensional if it has a base consisting of clopen sets). Hint: use certain Cantor scheme.

Zad. 8 (Alexandrov-Urysohn theorem) Show that ω^ω is the unique (modulo homeomorphism) nonempty, Polish, zerodimnesional space for which all compact sets have empty interior.