DST 3 2024

Zad. 1 Let X be an uncountable Polish space. Show that if $A \subseteq X$ has Baire Property and is not meager, then it has Perfect Set Property (i.e. it contains a perfect set).

Zad. 2 We say that a function $f\colon X \to Y$ is *Baire measurable* if $f^{-1}[U]$ has Baire property for each open $U \subseteq Y$. Suppose that X, Y are Polish and $f\colon X \to Y$ is Baire measurable. Show that there is a comeager set G such that $f|G$ is continuous.

Zad. 3 Prove that there is a set $A \subseteq 2^\omega$ such that 1) if $x \in A$ and x' differs from x on one bit, then $x' \notin A$, 2) if $x \notin A$ and x' differs from x on one bit, then $x' \in A$. (Recall that the Banach Mazur game for this set is not determined).

Zad. 4 Let X be a Polish space and let $A \subseteq X$. Show that if Adam has winning strategy in the Banach Mazur game for A, then A is a meager in a nonempty open set. (The proof on the lecture assumed additionally that A has Baire property). Hint: fix a winning strategy σ for Adam and let $s = (U_0, V_0, \ldots, U_n)$ be a run of the game played according to σ. Let

$$M_s = \{x \in U_n\colon \text{ for any move } V_n \text{ we have } x \notin U_{n+1} \text{ if } U_{n+1} \text{ is played according to } \sigma\}.$$

First, notice that for each $x \in A \cap U_0$ there is s (a run of the game suggested by σ) such that $x \in M_s$. Second, show that M_s is meager for each s as above.