DST 5 2024

Zad. 1 Let X be a Polish space, $A \subseteq X$, let $\Gamma(A)$ be the PSP game. Finally, let $T \subseteq \omega^{<\omega}$ be the tree associated to this game. Define $g\colon [T] \to X$ as the function which assigns the unique element of $\bigcap U^n_{i_n}$ to the run of the game

$$((U^0_0, U^0_1), i_0, (U^1_0, U^1_1), i_1, \dots)$$

interpreted as a branch in T. Show that g is continuous. Notice that the analogous function for the Banach-Mazur game (where the players move by sets of vanishing diameters, whose closures are contained in the previous moves) is also continuous.

Zad. 2 Let $A \subseteq X$ be analytic. We may define the *unfolded* version of the Banach-Mazur game for A. Fix a closed set $F \subseteq X \times \omega^\omega$ such that $\pi[F] = A$ and order Eve to play natural numbers additionally to her usual moves. Eve wins if $\langle x, y \rangle \in F$, where $x \in X$ is the unique element of the intersection of her moves and $y \in \omega^\omega$ is the sequence of natural numbers she played along the game. Show that if Eve has the winning strategy then A is comeager. Show that if Adam has the winning strategy, then A is meager in a non-empty open set.

Zad. 3 Conclude from the above that analytic sets are Baire measurable.

Zad. 4 Suppose that (A_n) is a sequence of analytic sets and $\bigcap_n A_n = \emptyset$, then there is a sequence (B_n) of Borel sets such that $A_n \subseteq B_n$ for each n and $\bigcap_n B_n = \emptyset$.

Zad. 5 Show that all analytic and coanalytic sets on 2^ω are Lebesgue measurable. (Hint: Lebesgue measurable sets are sets that differ from a G_δ on a set of measure zero. Use the Suslin operation.)

Zad. 6 Show that the Suslin operation is idempotent, i.e. $\mathcal{A}\mathcal{A}\Gamma = \mathcal{A}\Gamma$, where Γ a family of sets, and $\mathcal{A}\Gamma$ is the family of all the results of Suslin operation on a Suslin schemes built using elements of Γ.