Forcing **0** 2023

Zad. 1 Check that the following expressions are Δ_0:

- $x \subseteq y$,
- x is linearly ordered by $\in$,
- x is an ordinal number (hint: use the Axiom of Foundation),
- x is a natural number,
- $x = \omega$,
- $|x| = m$ (where m is a fixed natural number),
- f is a function,
- $z = \mathrm{dom}(f)$,
- $x = y \times z$.

Zad. 2 Is the expression

- x is finite

Δ_0? Is it absolute?

Zad. 3 Recall that a partial order $\mathbb{P}$ is non-atomic if for each $p \in \mathbb{P}$ there are $r, q \leq p$ such that $r \perp q$. We say that a partial order $\mathbb{P}$ is *separative* if

- for each $p \in \mathbb{P}$ there is $q < p$ and
- whenever $p \not\leq q$ there is $r \leq p$ such that $r \perp q$.

Investigate the relation between non-atomicity and separativeness.

Zad. 4 Suppose that $\mathbb{P}$ has the smallest element. What is $V[G]$?

In what follows $\mathbb{P}$ is a (separative) partial order, V is the ground model and G is a $\mathbb{P}$-generic over V.

Zad. 5 Prove that $V[G]$ satisfies the axiom of extensionality, union and foundation.

Zad. 6 Show that whenever $\dot{x}$ is a $\mathbb{P}$-name, then $\mathrm{rk}(\dot{x}_G) \leq \mathrm{rk}(\dot{x})$.

Zad. 7 How many $\mathbb{P}$-names $\dot{x}$ you can produce with the property: $\dot{x}_G = \emptyset$ for each generic G? How many with the property $\dot{x}_G = \{\emptyset\}$ for each G?