

Zad. 1 Check that the following expressions are Δ_0 :

- $x \subseteq y$,
- x is linearly ordered by \in ,
- x is an ordinal number (hint: use the Axiom of Foundation),
- x is a natural number,
- $x = \omega$,
- $|x| = m$ (where m is a fixed natural number),
- f is a function,
- $z = \text{dom}(f)$,
- $x = y \times z$.

Zad. 2 Is the expression

- x is finite

Δ_0 ? Is it absolute?

Zad. 3 Recall that a partial order \mathbb{P} is non-atomic if for each $p \in \mathbb{P}$ there are $r, q \leq p$ such that $r \perp q$. We say that a partial order \mathbb{P} is *separative* if

- for each $p \in \mathbb{P}$ there is $q < p$ and
- whenever $p \not\leq q$ there is $r \leq p$ such that $r \perp q$.

Investigate the relation between non-atomicity and separativeness.

Zad. 4 Suppose that \mathbb{P} has the smallest element. What is $V[G]$?

In what follows \mathbb{P} is a (separative) partial order, V is the ground model and G is a \mathbb{P} -generic over V .

Zad. 5 Show that whenever \dot{x} is a \mathbb{P} -name, then $\text{rk}(\dot{x}_G) \leq \text{rk}(\dot{x})$.

Zad. 6 How many \mathbb{P} -names \dot{x} you can produce with the property: $\dot{x}_G = \emptyset$ for each generic G ? How many with the property $\dot{x}_G = \{\emptyset\}$ for each G ?