Forcing 1 2025

Zad. 1 Prove that if M satisfies Axiom of Choice, then $M[G]$ satisfies Axiom of Choice. Hint: it is enough to show that for each $\dot{x}_G \in M[G]$ there is an ordinal number α and a function f (in $M[G]$) such that $\mathrm{dom}(f) = \alpha$ and $\dot{x}_G \subseteq \mathrm{rng}(f)$. Hint of the second order: well order $\dot{x}$ and let α be the order type.

Zad. 2 Show that a filter G is $\mathbb{P}$-generic iff it intersects every maximal antichain.

Zad. 3 Let $\mathbb{C}$ be the Cohen forcing. Suppose that $\dot{C}$ is a $\mathbb{C}$-name such that $1 \Vdash \dot{C} \subseteq \omega$. For each n let A_n be a maximal antichain of conditions deciding if $n \in C$.

$$\dot{B} = \{\langle n, p\rangle \colon n \in \omega, p \in A_n, p \Vdash n \in \dot{A}\}.$$

Show that

$$1 \Vdash \dot{A} = \dot{B}.$$