Forcing 2 2023

Zad. 1 (Maximal Principle) Suppose $p \Vdash \exists \dot{x}\ \varphi(\dot{x})$. Then there is a name $\dot{x}$ such that $p \Vdash \varphi(\dot{x})$.

Zad. 2 Let $X \in V$. Suppose that $p \Vdash \exists x \in \check{X}\ \varphi(x)$. Notice that it may happen that there is no $r \in X$ such that $p \Vdash \varphi(\check{r})$. However, if $p \Vdash \exists x \in \check{X} \varphi(x)$, then there is $q \leq p$ and $r \in X$ such that $q \Vdash \varphi(\check{r})$.

Zad. 3 Mathias forcing $\mathbb{M}$ consists of the conditions of the form $\langle s, N \rangle$, where s is finite, N is infinite and $\max s < \min N$. The ordering is defined as in the case of Mathias-Prikry forcing. Show that the generic real M induced by $\mathbb{M}$ is unsplit, i.e. for each $X \in P(\omega) \cap V$ we have $X \subseteq^* M$ or $X \cap M =^* \emptyset$.

Zad. 4 Show that Mathias forcing (defined in the previous exercise) is not ccc.

Zad. 5 Prove that $\mathbb{M}(\mathcal{F})$, where $\mathcal{F}$ is generated by a countable family, does not add a dominating real.

Zad. 6 Let $\mathbb{P}$ be the poset for adding a dominating real considered on the lecture. Let $f \in \omega^\omega$ be the $\mathbb{P}$-generic real. Let $A = f^{-1}$[Even numbers]. What are the properties of A? (Splitting, unsplit, dominating?)