Forcing 3 2025

Zad. 1 Check that Mathias-Prikry forcing is ccc.

Zad. 2 Let $S \subseteq \omega_1$ be stationary. Find a forcing $\mathbb{P}$ such that in $M[G] \models$"there is a club in S".

Zad. 3 Mathias forcing $\mathbb{M}$ consists of the conditions of the form $\langle s, N \rangle$, where s is finite, N is infinite and $\max s < \min N$. The ordering is defined as in the case of Mathias-Prikry forcing. Show that the generic real M induced by $\mathbb{M}$ is unsplit, i.e. for each $X \in P(\omega) \cap V$ we have $X \subseteq^* M$ or $X \cap M =^* \emptyset$.

Zad. 4 Show that Mathias forcing (defined in the previous exercise) is not ccc.

Zad. 5 Prove that $\mathbb{M}(\mathcal{F})$, where $\mathcal{F}$ is generated by a countable family, does not add a dominating real.

Zad. 6 Let $\mathbb{P}$ be the poset for adding a dominating real considered on the lecture. Let $f \in \omega^\omega$ be the $\mathbb{P}$-generic real. Let $A = f^{-1}$[Even numbers]. What are the properties of A? (Splitting, unsplit, dominating?)

Zad. 7 (for lovers of model theory) Let T be a complete theory and let M, N be models of T. We say that M and N are twins if M and N are not isomorphic, but there is a forcing $\mathbb{P}$ which forces that they are. Find examples of twins.