Forcing **3** 2023

Zad. 1 Let $i \colon \mathbb{P} \to \mathbb{Q}$ be a dense embedding. Let H be a $\mathbb{Q}$-generic and let $G = i^{-1}[H]$. Show that G is a generic filter.

Zad. 2 Show that every countable (separative) forcing notion is forcing equivalent to the Cohen forcing.

Zad. 3 Let $\mathcal{A}$ be an infinite maximal pairwise almost disjoint family. Define a forcing which will add an infinite subset of ω which is almost disjoint with each element of $\mathcal{A}$. What kind of properties this forcing has?

Zad. 4 Consider the forcing by $\mathbb{P} = \mathcal{P}(\omega)/fin$ (so the Boolean algebra of equivalence classes of the relation $A \sim B \iff A \triangle B$ is finite). Let G be a $\mathbb{P}$-generic. Let $\mathcal{U}$ be an ultrafilter (in the extension) defined by $U \in \mathcal{U}$ is $[U]_\sim \in G$.

- Show that $\mathbb{P}$ is σ-closed.
- Show that $\mathcal{U}$ is a non-principal ultrafilter.
- Show that $\mathcal{U}$ is a selective ultrafilter, i.e. if we color $[\omega]^2$ by two colors, then there is $X \in \mathcal{U}$ which is homogenuous, i.e. all elements of $[X]^2$ have one color. (Hint: by σ-closedness every such coloring is in the ground model).

Zad. 5 Suppose that we have $M \subseteq N$ and $N \models \exists a \in \mathbb{R} \setminus M$. Show that whenever $N \models \mathbb{R} \cap M$ is measurable, then $N \models \mathbb{R} \cap M$ is null. (Hint: use Steinhaus theorem)