Forcing **4** 2023

Zad. 1 Let G be a $\mathbb{C}$-generic (where $\mathbb{C}$ is the Cohen forcing). Show an example of a new real in $V[G]$ which is not a Cohen real (i.e. which does not avoid all ground model meager sets).

Zad. 2 Let G be as above. Show an example of an open subsets of 2^ω, in $V[G]$, which is not coded in the ground model.

Zad. 3 Propose some coding for F_σ sets.

Zad. 4 Check if

$$[\![\exists n \in \omega\ \varphi(n)]\!] = \bigvee_n [\![\varphi(n)]\!],$$

$$[\![\neg\varphi]\!] = [\![\varphi]\!]^c$$

Zad. 5 Let G be as above and let c be the generic real. Let $A \subseteq 2^\omega$, $A \in \Sigma^0_\alpha$. Show that there is $A' \in V$, $A' \in \Sigma^0_\alpha$, $A' \subseteq 2^\omega \times 2^\omega$ such that

$$A'_c = A.$$

Hint: consider a universal Σ^0_α set U. In $V[G]$: let x be such that $U_x = A$. Find $f\colon D \to 2^\omega$ such that $f(c) = x$. Cook up A' using U and g.

Zad. 6 Show that Cohen forcing $\mathbb{C}$ adds $\mathfrak{c}$ many Cohen reals.

Zad. 7 Show that the random forcing $\mathbb{M} = \mathrm{Bor}(2^\omega)/\mathcal{N}$ is not σ-centered.

Zad. 8 Let $\mathbb{M}_{\omega_2} = \mathrm{Bor}(2^{\omega_2})_{/\lambda_{\omega_2}=0}$. Let G be $\mathbb{M}_{\omega_2}$-generic and assume that $V \models CH$. Show that $V[G] \models \mathfrak{c} = \omega_2$.

Zad. 9 Is the generic real of the random forcing splitting?