Forcing **7** 2023

Zad. 1 Show that the iteration of κ many Cohen forcings is isomorphic to the product of κ many Cohen forcings. Show that they are isomorphic to $\mathbb{C}_\kappa$.

Zad. 2 Prove that $\mathbb{B} * \mathbb{B}$ does not add Cohen reals. (In particular $\mathbb{B} * \mathbb{B}$ and $\mathbb{B} \times \mathbb{B}$ are not equivalent).

Zad. 3 Show that if $\mathbb{P}$ is σ-closed and $1 \Vdash \dot{Q}$ is σ-closed, then $\mathbb{P} * \dot{\mathbb{Q}}$ is σ-closed.

Zad. 4 Let $\mathbb{M}$ be the Mathias forcing and let $\mathbb{M}_{\mathcal{U}}$ be the Mathias forcing parametrized by an ultrafilter. Prove that $\mathbb{M}$ is equivalent to $\mathcal{P}(\omega)/\mathrm{Fin} * \mathbb{M}_{\dot{\mathcal{U}}_{gen}}$ where $\dot{\mathcal{U}}_{gen}$ is the generic filter added by $\mathcal{P}(\omega)/\mathrm{Fin}$.

Zad. 5 Prove that finite support (infinite) iterations of non-trivial forcing notions add Cohen reals.

Zad. 6 Prove that full support (infinite) iterations of non-trivial forcing notions are not ccc.

Zad. 7 Let $\mathbb{P}_\omega = \langle \mathbb{P}_n, \dot{\mathbb{Q}}_n : n \in \omega \rangle$ be the finite support iteration of non-c.c.c forcings (i.e. $1_{\mathbb{P}_n} \Vdash$ "$\dot{Q}_n$ is not c.c.c. "). Prove that $\mathbb{P}_\omega$ collapses ω_1.

Zad. 8 Let $\mathbb{P}_{\omega_1} = \langle \mathbb{P}_\alpha, \dot{\mathbb{Q}}_\alpha : \alpha < \omega_1 \rangle$ be the countable support iteration of non-trivial forcings. Prove that $1_{\mathbb{P}_{\omega_1}} \Vdash$ "CH".
(extra) Prove that $1_{\mathbb{P}_{\omega_1}} \Vdash$ "$\diamondsuit$".