Applications of generic two-cardinal combinatorics

Piotr Koszmider

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P. Koszmider; On constructions with 2-cardinals. Math arxiv.

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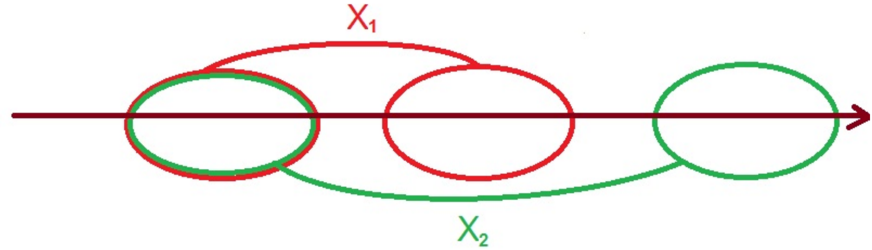
3. $ordtp(X) = ordtp(Y)$, then $f_{YX} : X \to Y$ - the unique order preserving bijection.
4. X_1, X_2 sets of ordinals of the same order type: amalgamation $X_1 * X_2 = X_1 \cup X_2$ if $X_1 \cap X_2 < X_1 \setminus X_2 < X_2 \setminus X_1$.




κ+

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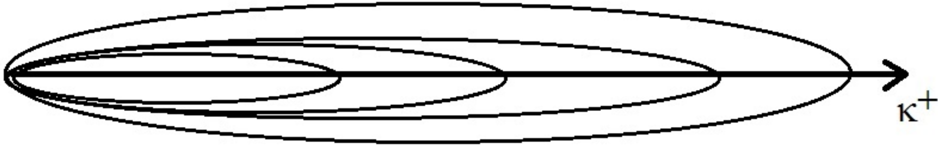
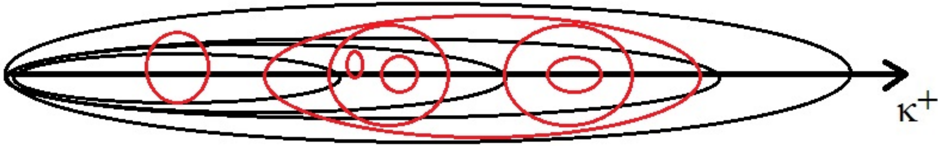


$\kappa^+ \subset P_{\kappa^+}(\kappa^+)$ $\quad\quad$ $\mu \subset P_\kappa(\kappa^+)$



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7. μ is *neat*, that is for every element X of μ of nonzero rank we have

$$X = \bigcup (\mu|X).$$

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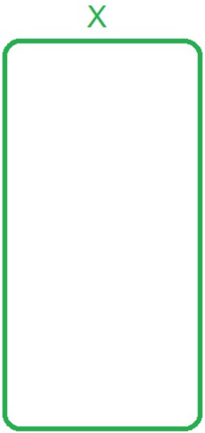

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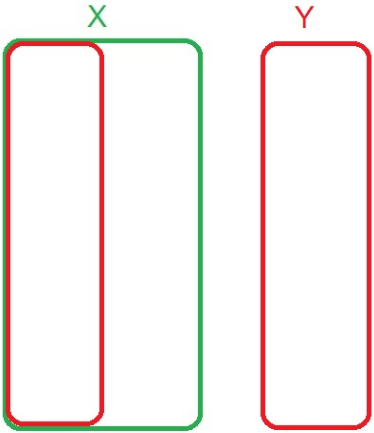

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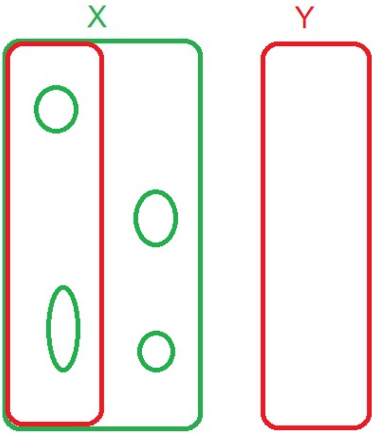

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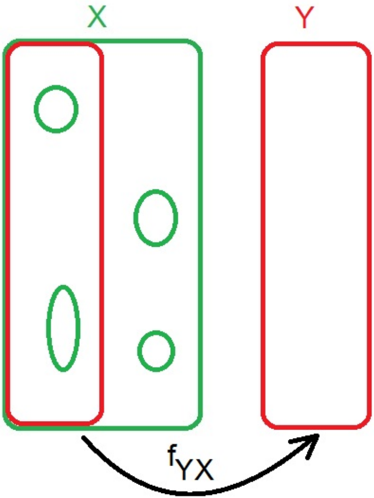

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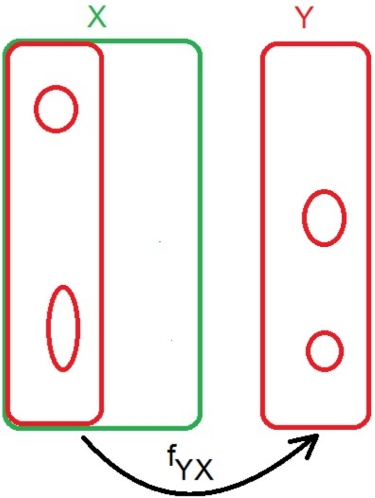

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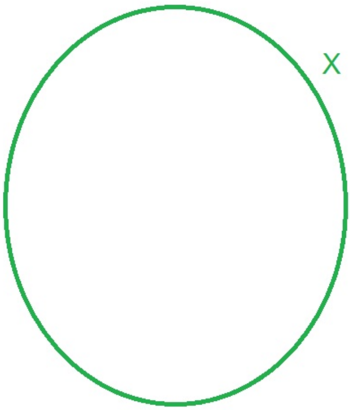

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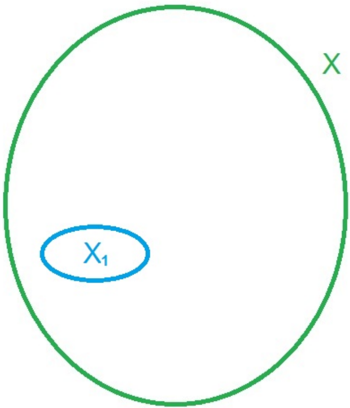

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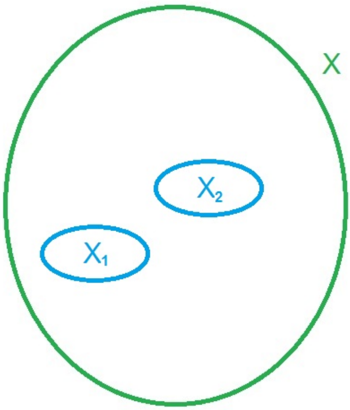

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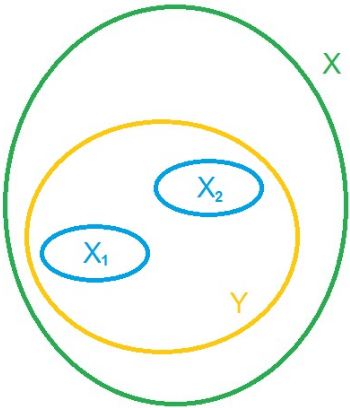

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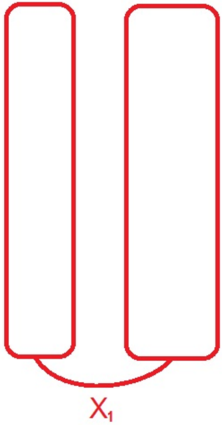

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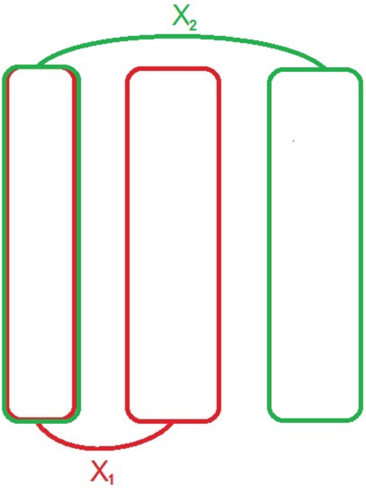

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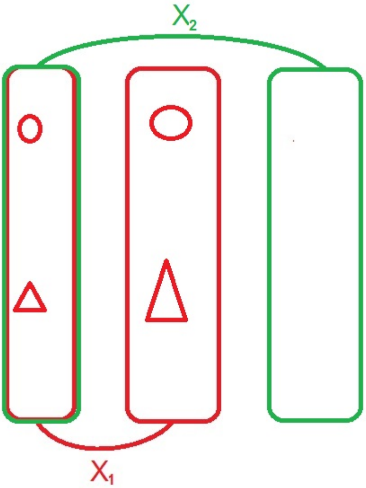

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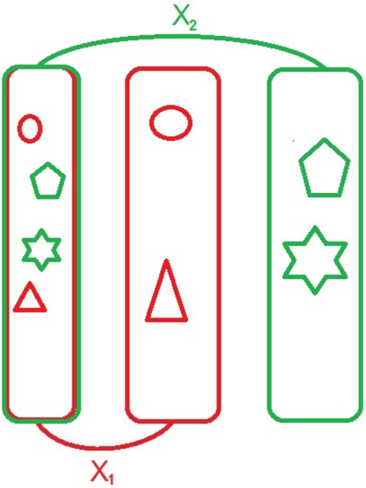

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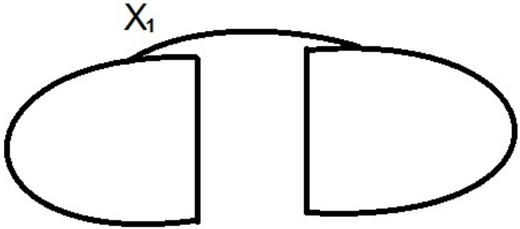

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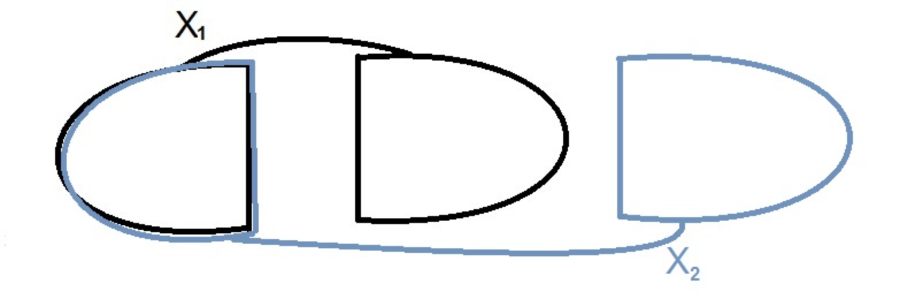

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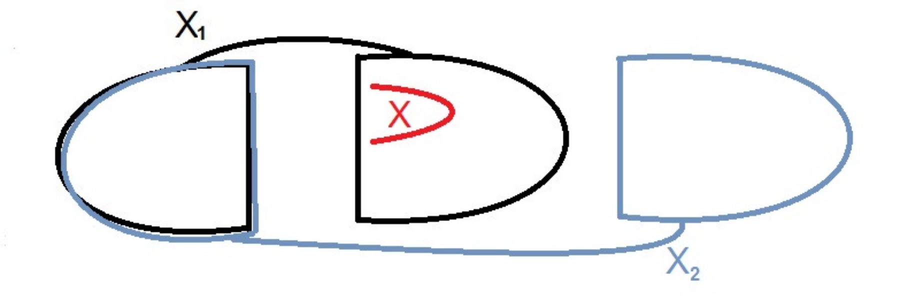

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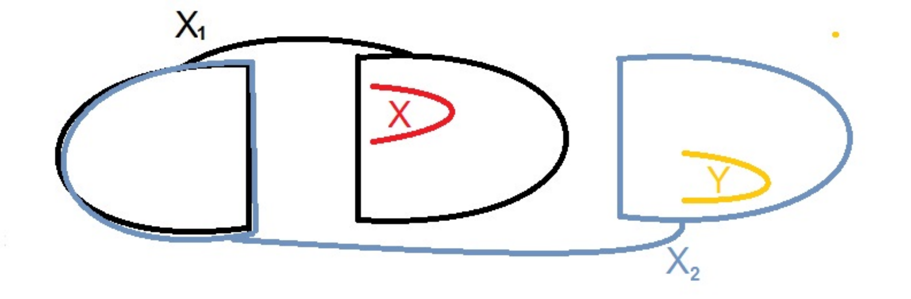

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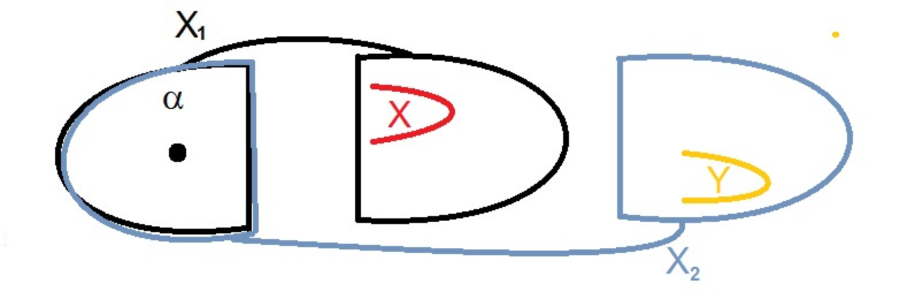

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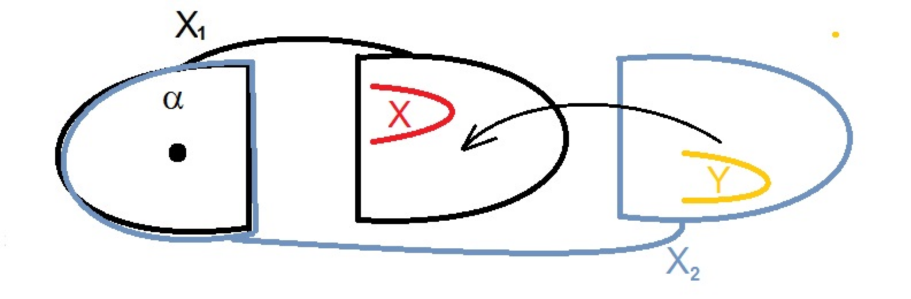

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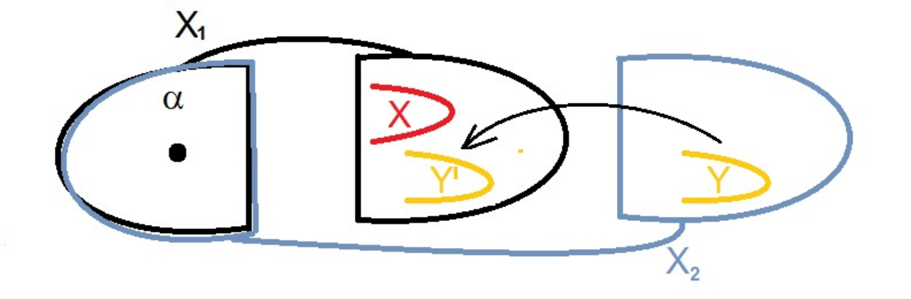

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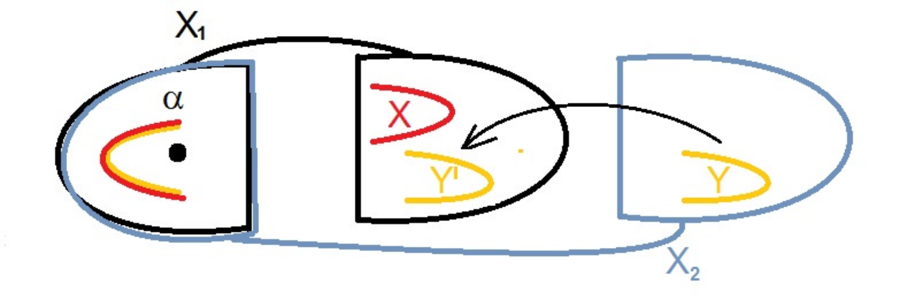

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Lemma (The coherence lemma)

Let κ be a regular cardinal and μ be a (κ, κ^+)-cardinal. Let $X, Y \in \mu$ be of the same rank and let $\alpha \in X \cap Y$, then

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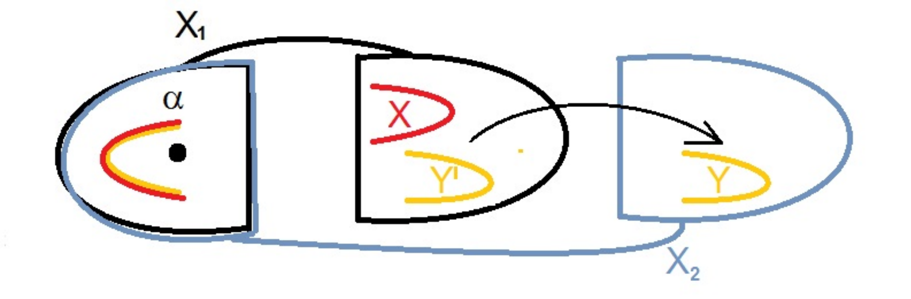

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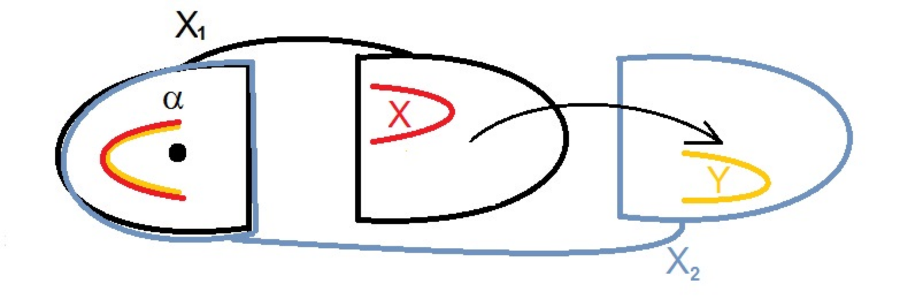

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Let κ be a regular cardinal and μ be (κ, κ^+)-cardinal. Every element $\alpha \in \kappa^+$ is in some $X \in \mu$ of any rank.

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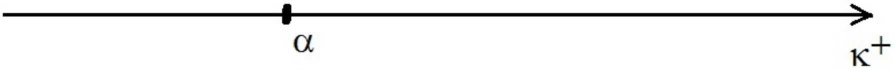

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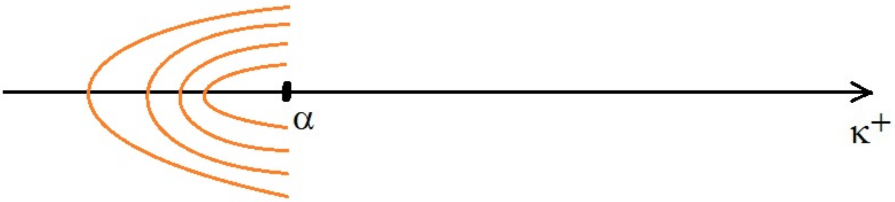

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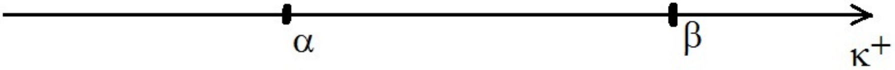

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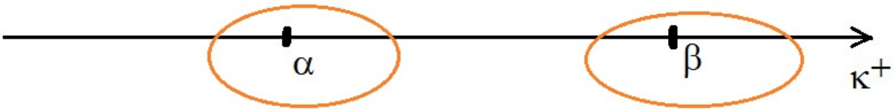

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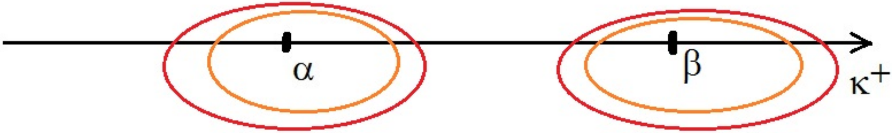

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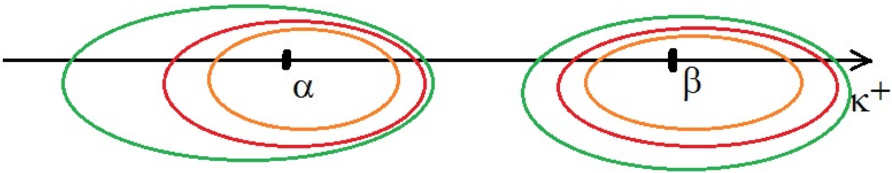

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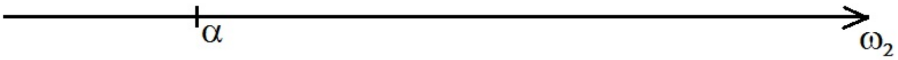

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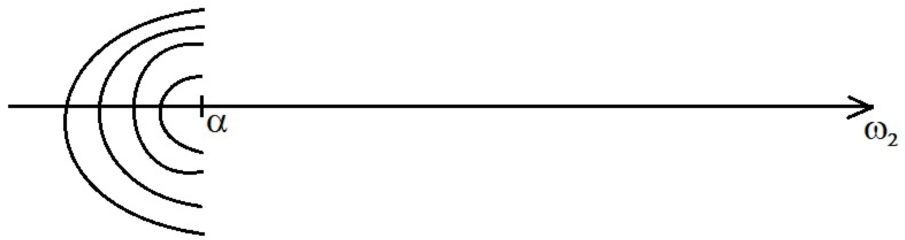

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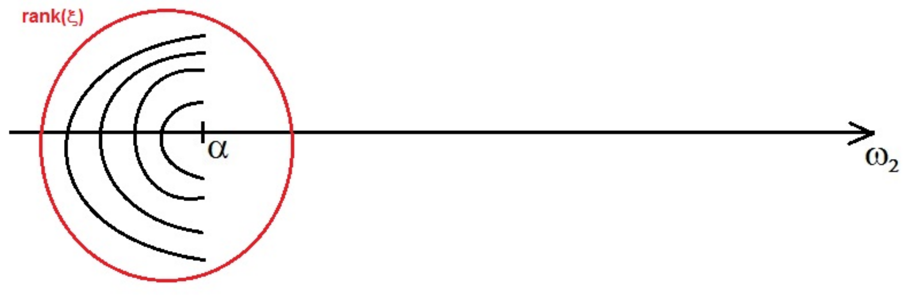

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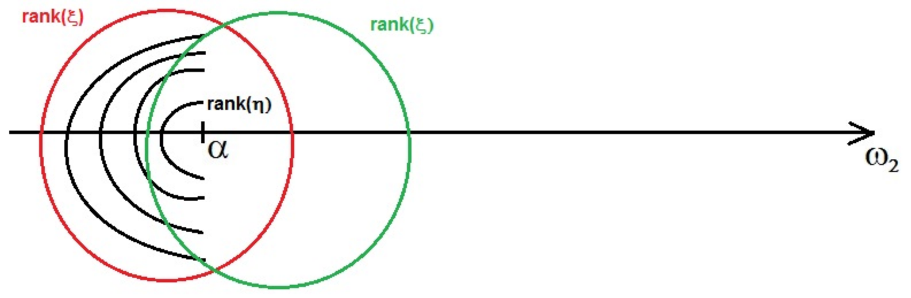

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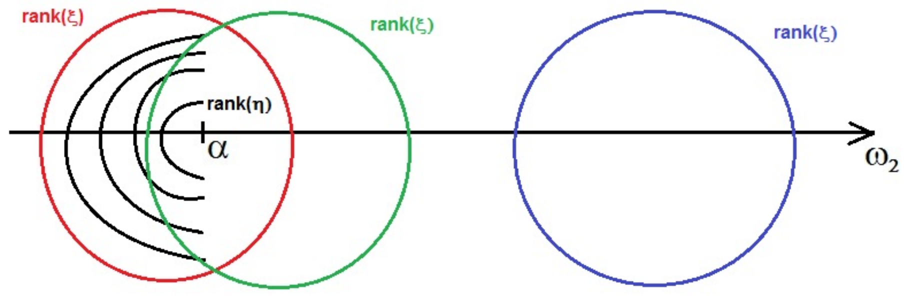

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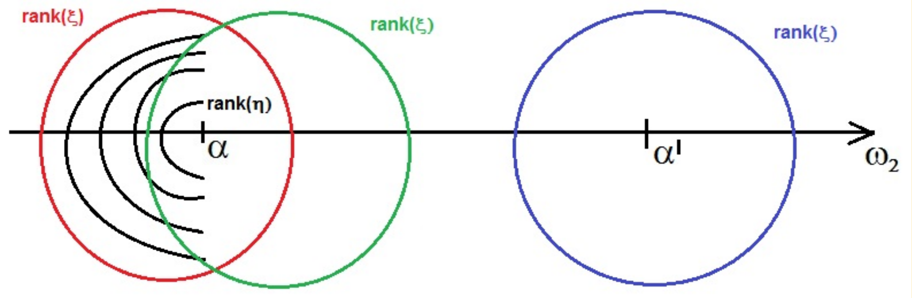

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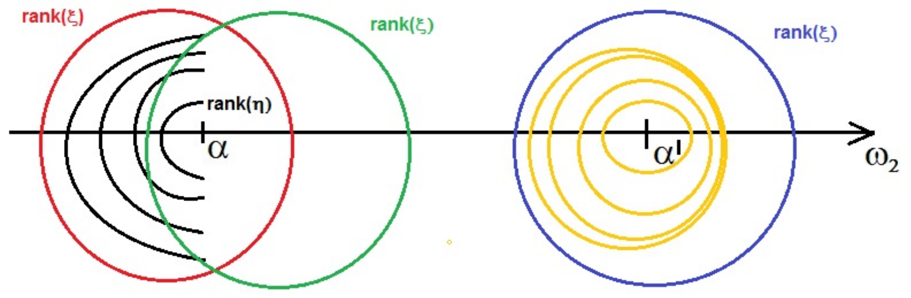

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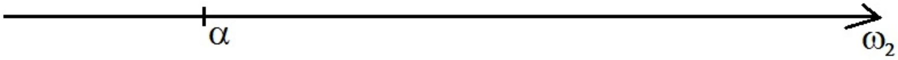

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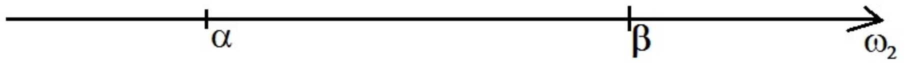

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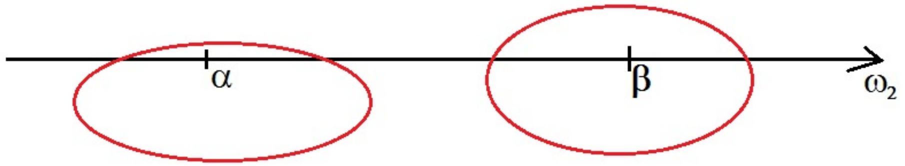

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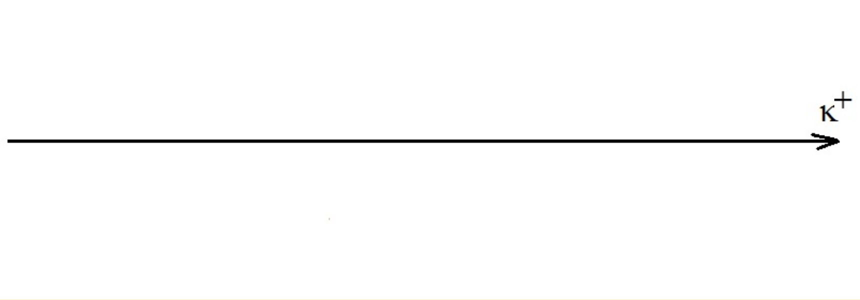

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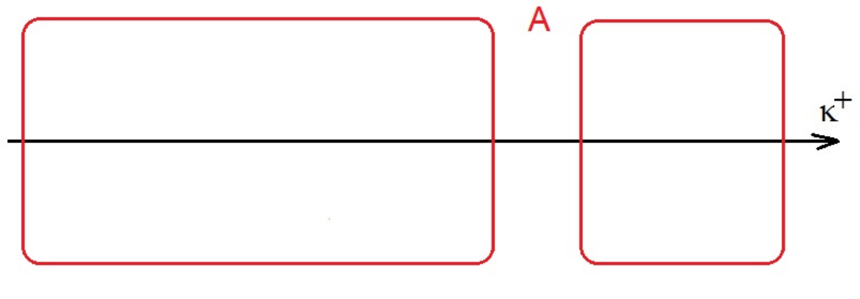

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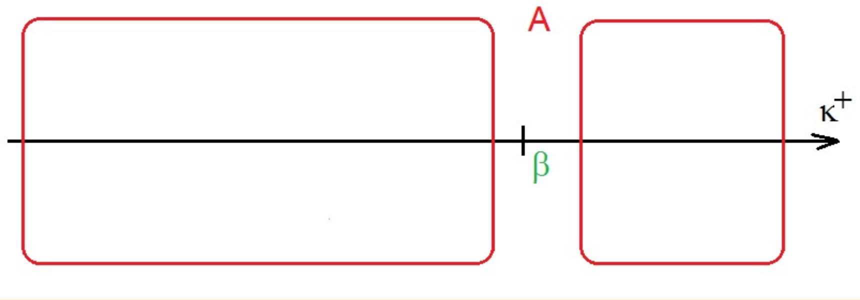

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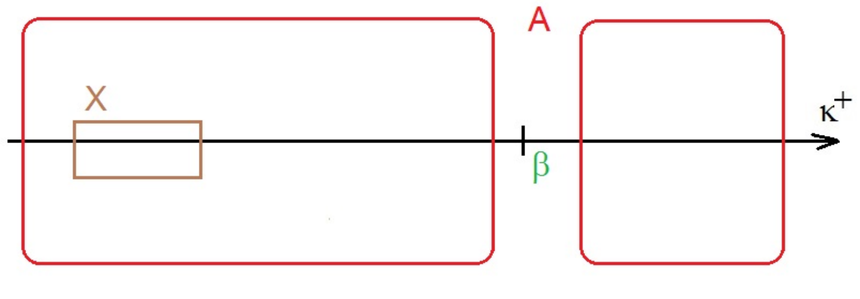

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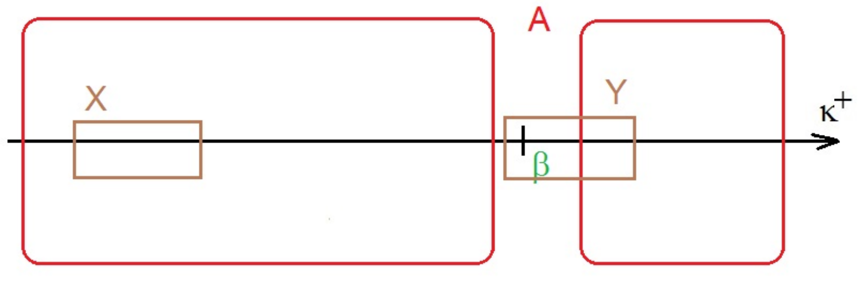

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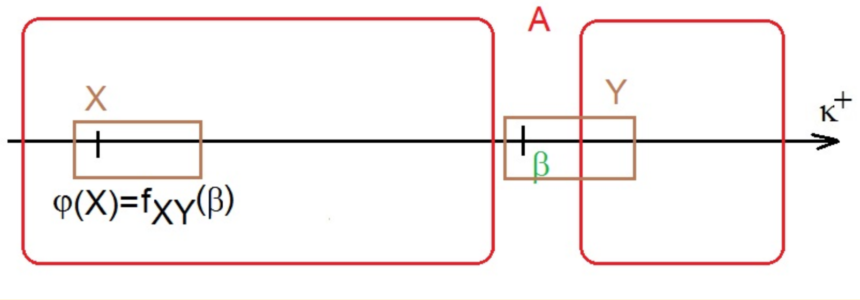

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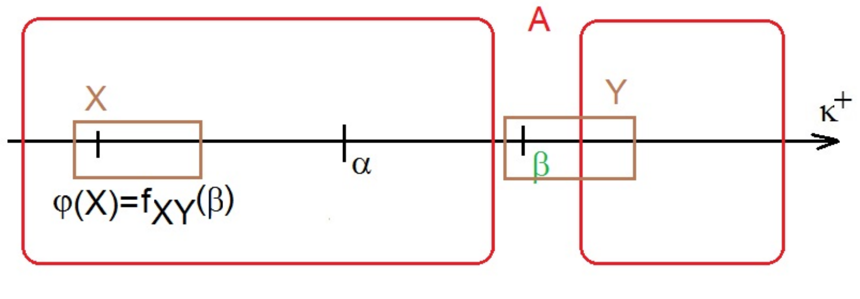

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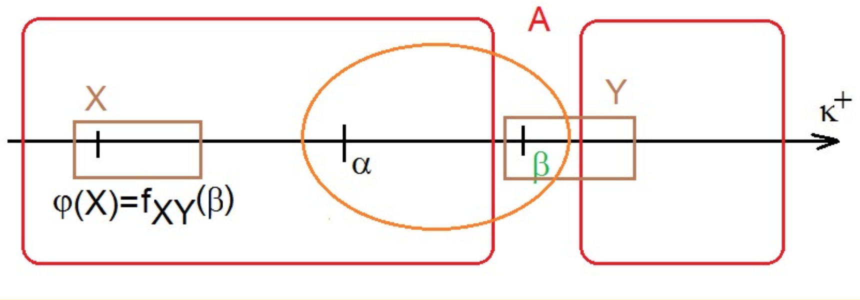

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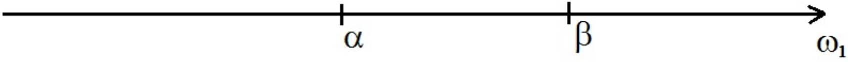

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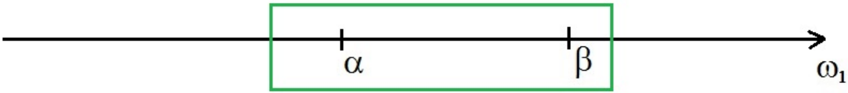

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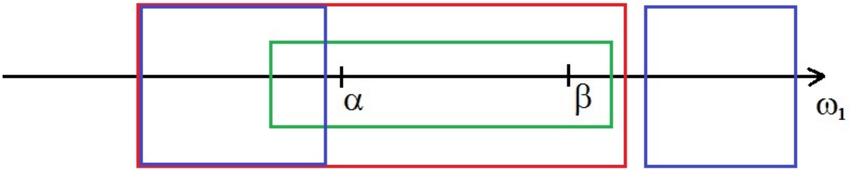

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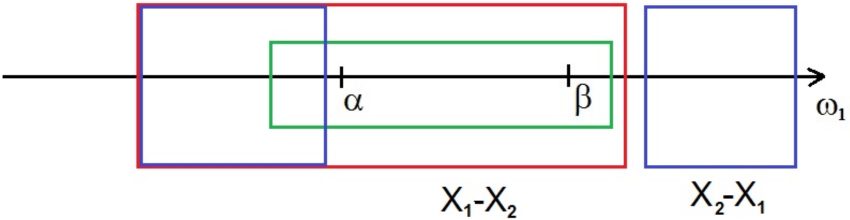

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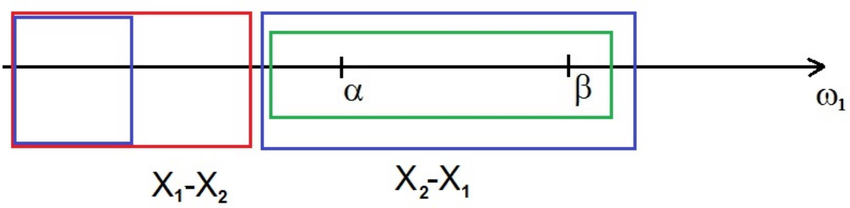

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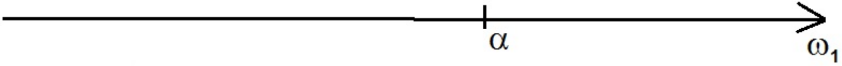

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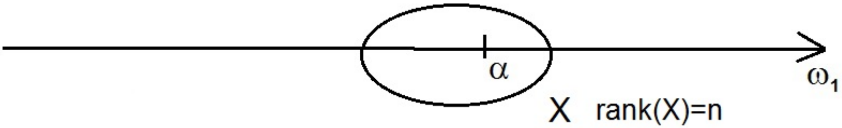

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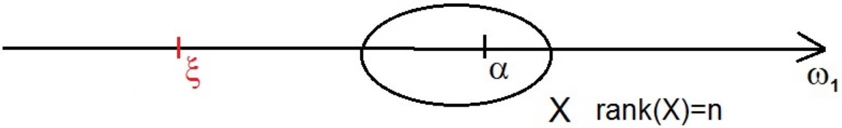

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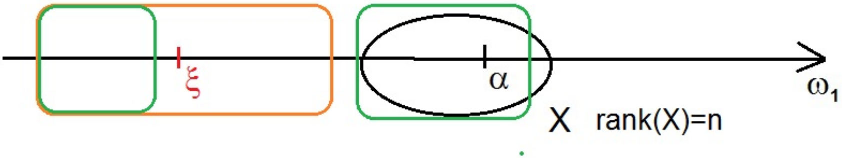

Definition

Let μ be a (κ, κ^+)-cardinal, then the following function $m_\mu = m\colon [\kappa^+]^2 \to \kappa$ is called a μ-coloring:

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Let $\alpha < \beta < \gamma < \kappa^+$, $\nu < \kappa$, $0 < \delta = \bigcup \delta < \epsilon < \kappa^+$, then the following conditions are satisfied:

(a) $|\{\xi < \alpha : \ m(\xi, \alpha) \leq \nu\}| < \kappa$

(b) $m(\alpha, \gamma) \leq \max\{m(\alpha, \beta), m(\beta, \gamma)\}$

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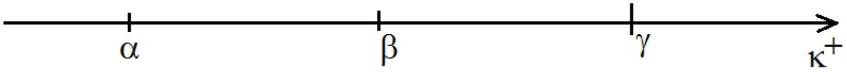

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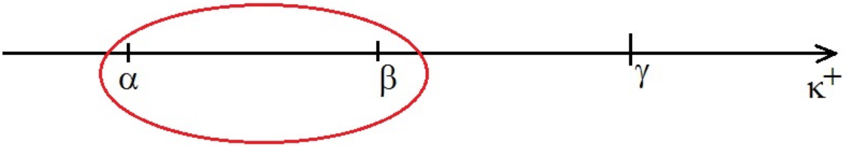

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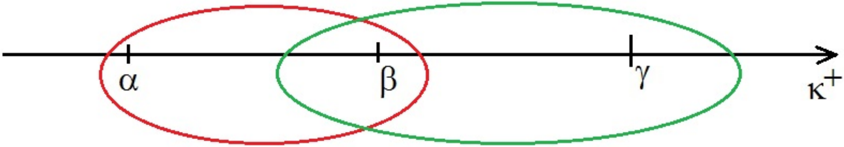

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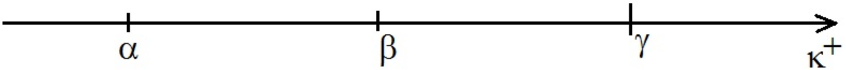

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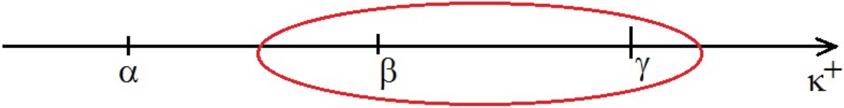

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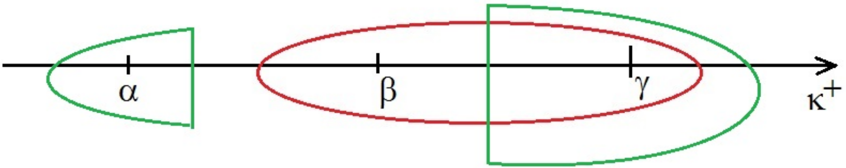

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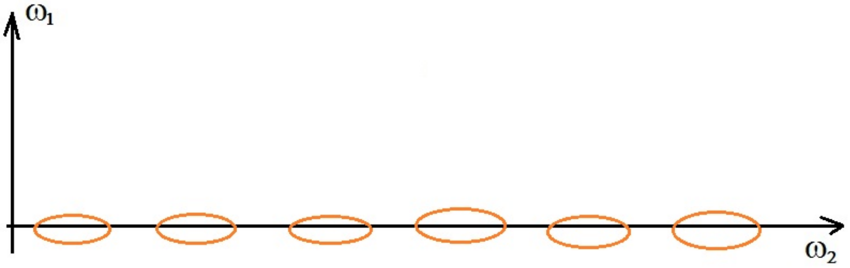

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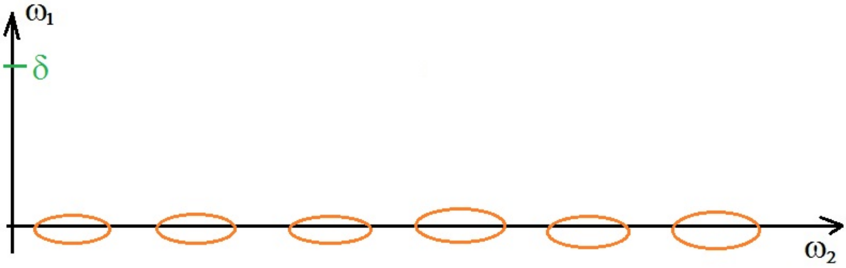

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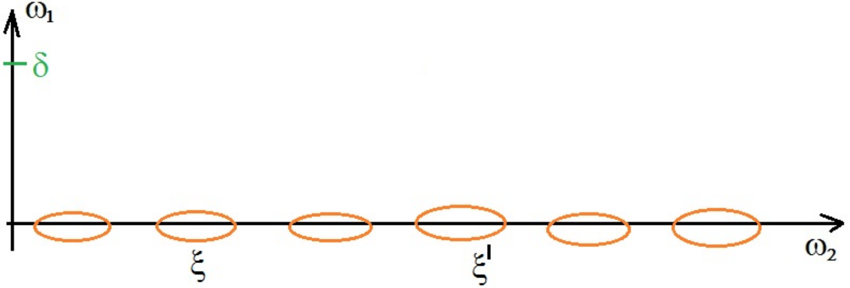

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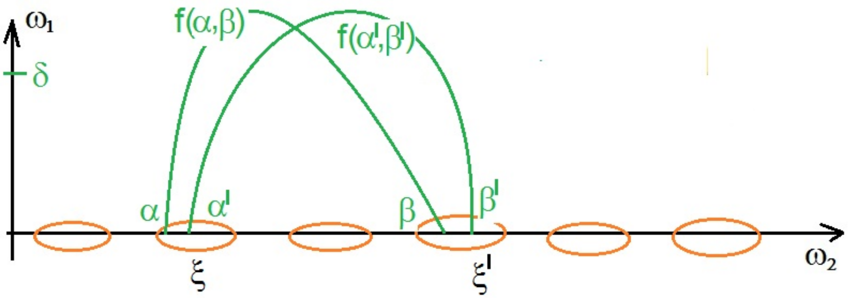

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Let $m : [\omega_2]^2 \to \omega_1$ be the μ-coloring. m is an unbounded function.

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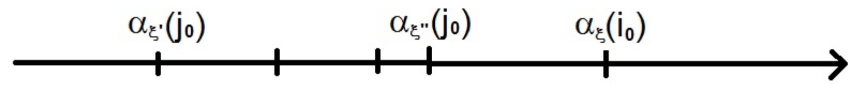

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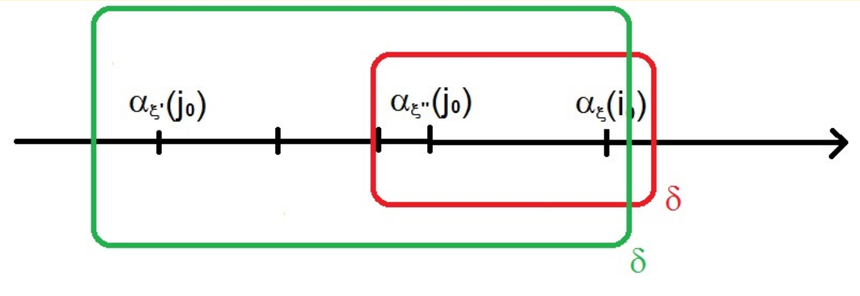

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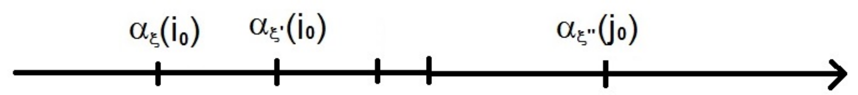

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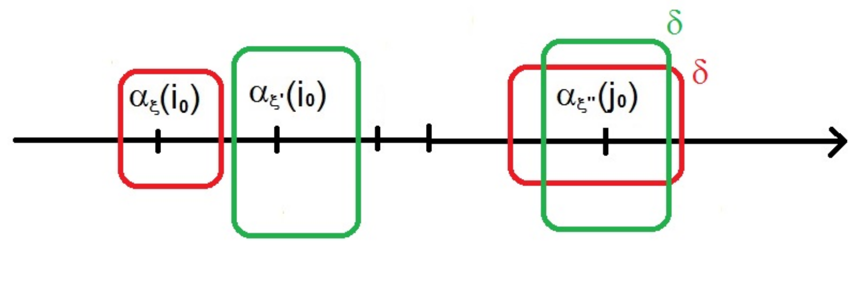

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Coherence lemma gives $X \cap M = M \cap \beta$.

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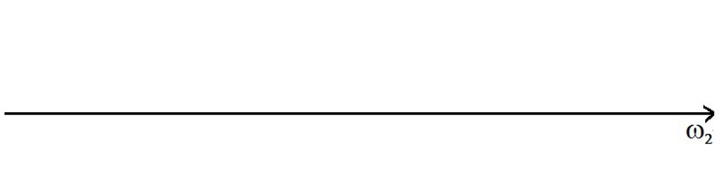

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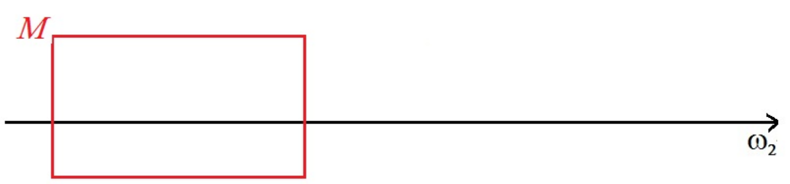

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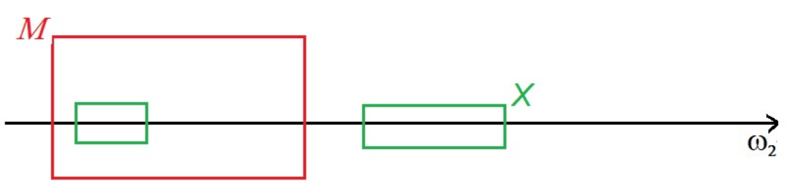

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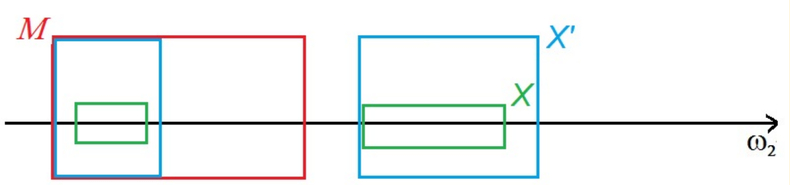

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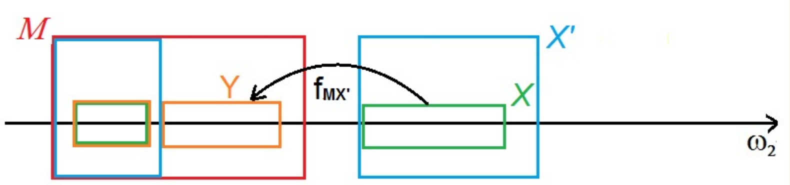


□

Definition

μ is *stationary coding set* iff μ is stationary subset of $[\omega_2]^\omega$ and that there is a one-to-one function $c : \mu \to \omega_2$ such that

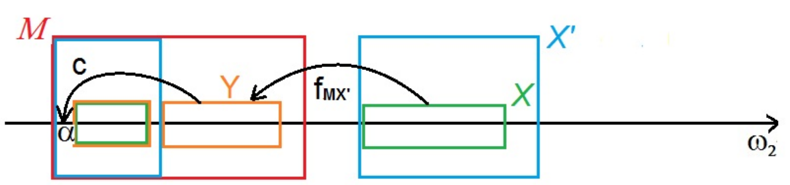
$$\forall X, Y \in \mu \;\; X \subset Y \;\Rightarrow c(X) \in Y.$$

Lemma

Suppose that an (ω_1, ω_2)-cardinal, $\mu \subseteq [\omega_2]^{\omega_1}$ is a stationary coding set and $\mu \in M \prec H(\omega_3)$, $|M| = \omega$, $M \cap \omega_2 \in \mu$ and $X \in M$. Then either $X \cap M = M \cap \beta$ for some $\beta \in \omega_2$ or $X \cap M \in M$.

Proof.

Case 2. $rank(X) < rank(M \cap \omega_2)$.



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