Contents

LICS-LC Long Lectures

Combinatorics of proofs

Martin Hyland

Ideally interpretations of proofs should exhibit some essential combinatorial features in an interesting and appealing way. As a case study, one can consider the notion of innocent strategy which is the basis for a game semantical interpretation of proofs and programmes. Some combinatorial content of this notion is sketched in the joint LICS paper accompanying this talk, whose abstract reads as follows.

We show how to construct the category of games and innocent strategies from a more primitive category of games. On that category we define a comonad and monad with the former distributing over the latter. Innocent strategies are the maps in the induced two-sided Kleisli category. Thus the problematic composition of innocent strategies reflects the use of the distributive law. The composition of simple strategies, and the combinatorics of pointers used to give the comonad and monad are themselves described in categorical terms. The notions of view and of legal play arise naturally in the explanation of the distributivity. The category-theoretic perspective provides a clear discipline for the necessary combinatorics.

There are other instances of a kind of categorical combinatorics of proofs, but in this talk I shall restrict myself to the one instance.

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Higher-order matching, games and automata

Colin Stirling

We describe a particular case where methods such as model-checking as used in verification are transferred to simply typed lambda calculus. Higher-order matching is the problem given $t = u$ where t, u are terms of simply typed lambda-calculus and u is closed, is there a substitution S such that tS and u have the same normal form with respect to beta eta-equality: can t be pattern matched to u? In the talk we consider the question: can we characterize the set of all solution terms to a matching problem? We provide an automata-theoretic account that is relative to resource: given a matching problem and a finite set of variables and constants, the (possibly infinite) set of terms that are built from those components and that solve the problem is regular. The characterization uses standard bottom-up tree automata. However, the technical proof uses a game-theoretic characterization of matching.

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LICS-LC SHORT LECTURES

Can logic tame systems programs?

Cristiano Calcagno

We report on our experience on designing and implementing tools for automatic reasoning about safety of systems programs using separation logic. We highlight some of the fundamental obstacles that need to be overcome, such as the complexity of data structures and scalability of the methods, on the path to realistic systems programs.

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Infinite sets that admit exhaustive search

Martin Escardó

Perhaps surprisingly, there are infinite sets that admit mechanical exhaustive search in finite time. We investigate three related questions: (1) What kinds of infinite sets admit exhaustive search? (2) How do we systematically build such sets? (3) How fast can exhaustive search over infinite sets be performed?

We give answers to them in the realm of Kleene–Kreisel higher-type computation: (1) involves the topological notion of compactness, (2) amounts to the usual closure properties of compact sets, including the Tychonoff theorem, (3) provides some fast algorithms and a conjecture.

These two talks include my contributed LICS paper, but go beyond in two respects: a general introduction to the role of topology in computation is given, and a few new results are included, such as an Arzela–Ascoli type characterization of exhaustible sets.

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Skolemization in constructive theories

Rosalie Iemhoff

It has long been known that Skolemization is sound but not complete for intuitionistic logic. We will show that by slightly extending the expressive power of the logic one can define a translation that removes strong quantifiers from predicate formulas and that is related but not equal to Skolemization. Since the extended logic is constructive, the translation can be considered as an alternative to Skolemization for constructive settings. The result easily implies an analogue of Herbrand's theorem. We will apply the method to various constructive theories and compare it to other Skolemization methods and related translations like the Dialectica Interpretation.

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Non-well-founded proofs

Alex Simpson

I will discuss various situations, arising in computer science, mathematics and logic, in which one is naturally led to consider associated proof systems involving interesting forms of non-well-founded proof.

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Tutorials

Cardinal Arithmetic in $L(R)$

Steve Jackson

In this series of talks we will survey the cardinal structure of the model $L(R)$ assuming the axiom of determinacy. We describe the close relationship between the cardinal structure and partition properties of the odd projective ordinals. We will present some recent simplifications to the presentation of this theory, as well as a result connecting the cardinal structure of $L(R)$ to that of the background universe V. We will attempt to make the talks as self contained as possible.

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Automatic structures

Bakhadyr Khoussainov

We study automatic structures. These are infinite structures that have automata presentations in a precise sense. By automata we mean any of the following: finite automata, tree automata, Büchi automata and Rabin automata.

Automatic structures possess a number of interesting algorithmic, algebraic and model-theoretic properties. For example, the first order theory of every automatic structure s decidable; automatic structures are closed under the first order interpretations; also, there are characterizations theorems for automatic well-founded partially ordered sets, Boolean algebras, trees, and finitely generated groups. Most of these theorems have algorithmic implications. For instance, the isomorphism problem for automatic Boolean algebras is decidable.

The first lecture covers basic definitions and presents many examples. We explain the decidability theorem that describes extensions of the FO logic in which each automatic structure has a decidable theory. The second lecture surveys techniques for proving whether or not a given structure can be presented by automata. We also talk about logical characterizations of automatic structures. The last lecture concentrates on complexities of automatic structures in terms of well-known concepts of logic and model theory such as heights of well-founded relations, Scott ranks of structures, and Cantor–Bendixson ranks of trees.

Most of the results are joint with Liu, Minnes, Nies, Nerode, Rubin, Semukhin, and Stephan.

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The infinitesimal subgroup of a definably compact group

Ya'acov Peterzil

Consider the compact Linear group $G = \mathrm{SO}(3, \mathbb{R})$. When G is viewed in any nonstandard real closed field, the set G^{00} of all matrices in G which are infinitesimally close to the identity forms a normal subgroup. Endow the quotient G/G^{00} with a "logic topology", whose closed sets are those whose preimages in G are type-definable. It is easy to see that G/G^{00}, with this logic topology, is isomorphic to $\mathrm{SO}(3, \mathbb{R})$, with the Euclidean topology.

Several years ago, A. Pillay conjectured that a similar phenomenon should be true for every "definably compact" group in an arbitrary o-minimal structure, even if the group itself was not defined over the real numbers. Roughly speaking, Pillay conjectured that every definably compact group G in a sufficiently saturated o-minimal structure has a canonical type-definable normal subgroup G^{00} such that the group G/G^{00}, when endowed with the logic topology as above, is isomorphic to a compact real Lie group. Moreover, the real dimension of this Lie group equals the o-minimal dimension of G.

My goal in these talks is to show, with the help of examples, how the interaction between different notions, such as o-minimality, Lie groups, compactness, measure theory, and Shelah's Independence property, yields a solution to the conjecture.

For background on o-minimality, see van den Dries's book [2] below. For Pillay's conjecture, see [6]. For key-steps in the solution to the conjecture, see [1, 5, 3, 4].

[1] A. Berarducci, M. Otero, Y. Peterzil, and A. Pillay, *A descending chain condition for groups in o-minimal structures*, ***Annals of Pure and Applied Logic***, vol. 134 (2005), pp. 303–313.

[2] L. v. d. Dries, ***Tame topology and o-minimal structures***, Cambridge University Press, New York, 1998.

[3] M. Edmundo and M. Otero, *Definably compact abelian groups*, ***Journal of Mathematical Logic***, vol. 4 (2004), pp. 163–180.

[4] E. Hrushovski, Y. Peterzil and A. Pillay, *Groups, measure and the NIP*, ***Journal of the American Mathematical Society***.

[5] Y. Peterzil and A. Pillay, *Generic sets in definably compact groups*, ***Fundamenta Mathematicae***, vol. 193 (2007), pp. 153–170.

[6] A. Pillay, *Type-definability, compact Lie groups, and -minimality*, ***Journal of Mathematical Logic***, vol. 4 (2004), pp. 147–162.

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Plenary Lectures

Structured finite model theory

Albert Atserias

Many of the classical results of model theory, most notably some of the direct consequences of the Compactness Theorem, fail badly in restriction to finite structures. A classical example is the Łoś-Tarski preservation-under-substructures Theorem, which is known to fail in the finite. Motivated by certain application-areas in computer science and combinatorics, it has been argued that it might be profitable to consider some further restrictions beyond finiteness. What classical theorems of model theory hold on classes of finite trees? or finite planar graphs? or finite structures of bounded treewidth? or, more generally, minor-closed classes of finite graphs? The goal of our talk is to give an overview of the recent results in this area. Interestingly, many of these results have at its heart an application of Gaifman's Locality Theorem, which seems to play in this area a role comparable to the one played by the Compactness Theorem in classical model theory.

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Towards a proof theory of analogical reasoning

Matthias Baaz

In this lecture we compare three types of analogies based on generalizations and their instantiations:

1. Generalization w.r.t. to invariant parts of proofs (e.g., graphs of rule applications etc.).

2. Generalization w.r.t. to an underlying meaning. (Here proofs and calculations are considered as trees of formal expressions. We analyze the well-known calculation of Euler demonstrating that the 5th Fermat number is compound.)

3. Generalization w.r.t. to the premises of a proof. (This type of analogies is especially important for juridical reasoning.)

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Computable analysis and effective descriptive set theory

Vasco Brattka

Computable analysis can be considered as an extension of computability theory to infinite objects such as real numbers, continuous functions and closed subsets that are studied in analysis. In a similar way as discrete degrees of non-computability can be classified in the arithmetical hierarchy, the corresponding hierarchy for such classifications in analysis is the Borel hierarchy. Thus, it is natural to expect a fruitful interaction between effective descriptive set theory and computable analysis. In this talk we present an extension of the Representation Theorem of computable analysis to effectively Borel measurable maps. This theorem allows to introduce a conservative extension of the notion of effective Borel measurability to admissibly represented topological spaces as they are used in computable analysis. A resulting notion of reducibility for functions is described and techniques of completeness proofs from computability theory can be imported into this branch of effective descriptive set theory. We apply these techniques to characterise the degree of non-computability of topological operations such as the closure, the interior, the boundary and the derived set operation with respect to hyperspace representations for computable metric spaces.

[1] Vasco Brattka, *Effective Borel measurability and reducibility of functions*, ***Mathematical Logic Quarterly***, vol. 51 (2005), no. 1 pp. 19–44.

[2] Vasco Brattka and Guido Gherardi, *Borel complexity of topological operations on computable metric spaces*, ***Computation and Logic in the Real World***, (S. B. Cooper, B. Löwe, and A. Sorbi, editors), Lecture Notes in Computer Science, vol. 4497, Springer, Berlin, 2007.

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Model theory of difference fields, and some applications

Zoé Chatzidakis

A difference field is a field with a distinguished automorphism. I will first give a survey of the model theory of the existentially closed difference fields, and then of some application(s) to diophantine geometry. The main tool for applications is the dichotomy theorem, and I will spend some time explaining what are its consequences in the particular context of difference fields, and how they can be used.

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Coding compact spaces of Borel functions

Gabriel Debs

Let $B(X)$ denote the space of all Borel functions on some Polish space X, equipped with the pointwise convergence topology (i.e., the topology induced by the product topology on $\mathbb{R}^X$). When X is uncountable the space $B(X)$ is clearly non metrizable. But it follows from the early work of Rosenthal and Bourgain–Fremlin–Talagrand that the **compact** subspaces of $B(X)$ possess many metric-like properties: For example most of the standard topological notions admit sequential formulations. Notice that such a compact space K needs not to be metrizable even if it is separable. However in this latter case, one can equip K with some natural "descriptive structures" that we shall describe in more detail in this talk. One consequence of the main result that we shall present here is that all these structures are actually "equivalent" in some sense. This will also provide a positive answer to a question asked by Argyros, Dodos and Kanellopoulos concerning the notion of **analytic** Rosenthal compacta that they introduced recently. Though many of the statements we shall consider are totally classical, the proofs make a crucial use of Effective Descriptive Set Theory, namely of some fundamental results of Moschovakis on inductive definability.

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On a new functional interpretation

Fernando Ferreira

Gödel's functional "Dialectica" interpretation of 1958 was published with the explicit aim of being a contribution to Hilbert's program. As things go, it introduced a new technical tool in Proof Theory: one that presents a trade-off between quantifier complexity and finite-type computable functionals. Gödel's particular interpretation reduces HA (Heyting arithmetic) to a quantifier-free calculus T of finite-type functionals, thereby reducing the consistency of the former to the consistency of the latter. It is seldom observed explicitly that Gödel's interpretation can also be used to prove certain conservation results. For instance, the adjunction of certain principles (axiom of choice AC, independence of premises IP, Markov's principle MP) does not change the provably Π^0_2-sentences of HA.

We describe a new (2005) functional interpretation of HA. This functional interpretation (due to the author and Paulo Oliva) uses the same functionals as Gödel's but changes the assignment of formulas. Contrary to Gödel's assignment, our assignment does not preserve set-theoretic truth. The new interpretation yields new conservation results. Some false set-theoretic principles like the refutation of extensionality or Brouwer's FAN theorem, as well as some semi-intuitionistic principles like LLPO (less limited principle of omniscience) or weak König's lemma (WKL), can be adjoined to HA without changing the provably Π^0_2-sentences. More memorably, within intuitionistic logic, the above principles are not able to prove further terminating computations.

As an illustration, we show that the provably total functions of the classically inconsistent second-order **intuitionistic** theory $\mathrm{WKL}_0 + \mathrm{LLPO} + \mathrm{IP} + \mathrm{MP} + AC^{\mathbb{N}} + \mathrm{FAN}$ are the primitive recursive functions, where $\mathrm{AC}^{\mathbb{N}}$ is the countable axiom of choice. By the negative translation of Gödel–Gentzen, we get, as an easy corollary, Harvey Friedman's well-known conservation result of (classical) WKL_0 over RCA_0.

Finally, we would like to draw attention to an ubiquitous principle in mathematical logic: the bounded collection scheme. It appears in bounded theories of arithmetic, in admissible set theory (as Δ_0-collection), and in analysis (if we view Brouwer's FAN principle as a form of collection) and proof mining (Kohlenbach's uniform boundedness principles). We believe that the new interpretation provides a fresh and integrated look at bounded collection.

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Philosophical content of formal achievements

Andrzej Grzegorczyk

Two theses will be proposed:

The first general Thesis: A Perception of Philosophical Content depends on the **Taste** of the Scholar. In Science about reality it is **Truth**. This means a good **description** of real objects (events).

In Science about potentialities (formal/imaginary Science), the value is an **Invention** of Consistent Construction and may be called: **joke** and/or **beauty**.

The second Thesis: In formal science, called here **imaginaries**, we can make the following similar **distinction** in the **taste** of scholars:

- **Maths**: Mathematicians were always proud that they can **calculate** functions. They, e.g., **calculate by recursion**: $f(0) = a$, $f(n+1) = F(f, n) \ldots$
  Calculation is the fundamental joke (or beauty) of the construction.
- **Logic**: Logicians are more philosophers, and were always proud that they can **define** relations. They **define** using logical **connectives** and **quantifiers**.
  Definition is the fundamental joke (or beauty).

Some arguments, an example, and a propaganda for these distinctions will be given.

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Brownian motion and Kolmogorov complexity

Bjørn Kjos-Hanssen

Probabilistic potential theory contains results of the following form: A randomly chosen small set of reals is almost surely disjoint from a given small set of reals. Here the size of a set can be for example its Hausdorff dimension.

Using computability theory and Kolmogorov complexity, we obtain some results of this kind, for certain randomly chosen sets associated with Brownian motion. We then investigate to what extent these results can be reproduced using only potential theory.

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Definability in differential fields

Piotr Kowalski

A differential field is a field equipped with a derivation. In the positive characteristic case one often needs to "improve" the notion of derivation by introducing Hasse–Schmidt derivations. I will briefly discuss the first-order theory of (differentially closed) differential fields. Then, I will focus on two special kinds of definable sets there. Sets of the first kind played a crucial role in the Hrushovski's proof of the Mordell–Lang conjecture [2]. Sets of the second kind appear in the statement of a theorem of Ax [1] and its generalizations.

[1] James Ax, *On Schanuel's conjectures*, ***Annals of Mathematics***, vol. 93 (1971), no. 2, pp. 252–268.

[2] Ehud Hrushovski, *Mordell–Lang conjecture for function fields*, ***Journal of the American Mathematical Society***, vol. 9 (1996), pp. 667–690.

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Large cardinals and forcing-absoluteness

Paul B. Larson

Absoluteness theorems of Levy and Shoenfield state that the truth values of Σ_1 and Σ^1_2 sentences cannot be changed by set forcing. In the presence of large cardinals, forcing-absoluteness extends to much larger classes of statements. Jumping to the end of a long story, we will talk about some of the strongest forcing absoluteness results, including extensions of Woodin's Σ^2_1 absoluteness theorem.

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Sets and the concept of set

Donald A. Martin

Those who doubt that the subject matter of mathematics is a realm of abstract objects often do so because of a general disbelief in the existence of abstract objects. I will propose a very different basis for doubts about the status of mathematical objects. Many people think that various statements of set theory, e.g., the Continuum Hypothesis, are neither true nor false. I am not one of one of these people, but I do think we cannot at present be certain that the CH has a truth-value. I will argue that if it is not certain that the CH has a truth-value then it is also uncertain whether there is any system of objects that satisfies our concept of a universe of sets. It is common to say that the CH has no truth-value and to blame this on the existence of many systems of objects that satisfy the set concept. Part of my job will be to discredit this position. Another part will be to give an account of what set theoretic truth and falsity could amount to if there is no universe of sets.

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Special Session:

Logic and Analysis

Local stability of ergodic averages

Philipp Gerhardy

'Proof mining' is the subfield of mathematical logic that is concerned with the extraction of additional information from proofs—even ineffective proofs!—in mathematics and computer science. The main focus is on developing general (and feasible) methods to unwind the computational content of proofs and to apply these methods to real existing proofs. We present an application of proof mining in the field of ergodic theory. The Mean Ergodic Theorem states that a for a nonexpansive operator T on a Hilbert space H and for any f in H, the sequence of ergodic averages $A_n f := 1/n + 1\Sigma_{i \leq 0} T^i f$ converges to a limit. While a full rate of convergence is not possible, we present explicit rates for the classically equivalent statement that for any number-theoretic function K, the averages $A_m f$ are stable within ε in some interval $[n, K(n)]$. These computable bounds have been obtained by applying methods of proof mining to a standard textbook proof of the Mean Ergodic Theorem.

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Computability and non-computability results for the topological entropy of shift spaces

Peter Hertling

The topological entropy, a numerical quantity assigned to a continuous function from a compact space to itself, is invariant under topological conjugacy and serves as a tool for classifying dynamical systems. Therefore, computing the topological entropy is an important problem in dynamical systems theory. We discuss several recent positive and negative results concerning the computability of the topological entropy, mostly of shift dynamical systems.

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Geometry of the Urysohn space: a model-theoretic approach

Julien Melleray

The Urysohn space was built by Pavel Urysohn in 1924. Over the past 15 years or so, there has been growing interest in this space and its geometry. In the talk I'll try to explain how one can use model-theoretic methods (via the framework of model theory of metric structures) to tackle some problems involving the Urysohn space and its geometry. I'll discuss in particular the homogeneity properties of the space, and questions about conjugacy of isometries.

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Analytic Combinatorics of the transfinite

Andreas Weiermann

Roughly speaking, Analytic Combinatorics (AC) is the art of counting using Cauchy's integral formula. Typically (following Flajolet et al.) AC is used for investigations on the average case analysis of algorithms. In this talk we will show how to apply AC to systems of ordinal notations (for relatively small ordinals). We explain surprising connections to the theory of partitions as studied, for example, by Hardy and Ramanujan.

The resulting information is used for proving logical limit laws (joint work with Alan R. Woods) and for classifying phase transition thresholds for Gödel incompleteness.

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Special Session: Model Theory

Independence in structures and finite satisfiability

Vera Djordjevic

An outline of ideas and methods concerning proving the finite submodel property by probabilistic means will be given. It appears like some notion of "sufficient independence" between definable relations is needed for a probabilistic argument to work out. We consider a notion, the n-embedding of types property, which has some relationship with the n-amalgamation property considered elsewhere. The main result about the finite submodel property applies to countably categorical simple structures with trivial forking, finite SU-rank and with the n-embedding of types property for every n.

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Some thoughts on bad objects

Amador Martin-Pizarro

The Algebraicity Conjecture states that a simple 2^{ω}-categorical group can be seen as an algebraic group over an algebraically closed field. This long-standing open conjecture belong to a wider conjecture, called "Principe du Nirvana", whose various specific instances have been refuted in the last decades. A specific programme in order to characterize such simple groups was described, which originally imposed the non-existence of certain objects called bad fields (describing an undesired configuration characterizing the Borel subgroups). We will discuss the relevance of bad fields and foster diverse links with other areas of algebraic geometry.

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Countable imaginary simple unidimensional theories

Ziv Shami

We show that these theories are supersimple, provided that forking is witnessed by pseudo-low formulas (a formula is pseudo-low if forking by it is a type-definable relation). In particular, any low or 1-based countable imaginary simple unidimensional theory is supersimple.

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Super real closed rings

Marcus Tressl

A super real closed ring is a commutative ring A with unit together with functions $F_A\colon A^n \to A$ for all n in N and every continuous function $F\colon \mathbb{R}^n \to \mathbb{R}$, satisfying the composition rules $(F \circ (\dots, G, \dots))_A = F_A \circ (\dots, G_A, \dots)$. For example, every ring $C(X)$ of continuous real-valued functions is a super real closed ring, where $F_{C(X)}$ is composition with F; also super real fields in the sense of Dales–Woodin at prime z-ideals are naturally equipped with a super real closed ring structure.

I will overview the algebraic properties of super real closed rings, give an application to o-minimal structures expanding the real field and discuss the relation to C^∞–rings (in the sense of Moerdijk–Reyes).

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Special Session:

Philosophical and Applied Logic at the Journal of Philosophical Logic

From philosophical logic to computer science—and back

G. Aldo Antonelli

Over the past 25 years or so, computer scientists have looked at philosophical logic in search of tools for the formal modeling of a number of interactive computational processes, whereas philosophical logician have drawn new inspiration from a set of problems specific to theoretical computer science and artificial intelligence. This talk provides a (perforce idiosyncratic) survey of the interactions between the two fields, pointing out interdisciplinary connections and fruitful cross-fertilizations.

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Philosophical logic meets formal epistemology

Horacio Arlò Costa

Formal epistemology is a branch of epistemology that uses formal methods to articulate and solve traditional epistemic problems. The talk surveys some recent interactions between philosophical logic and formal epistemology. We focus on epistemic problems related to the representation of attitudes and their change, both for single and multiple agents. The consideration of these problems has motivated, for example, new work in areas like belief revision, epistemic and dynamic logic or formalisms to represent uncertainty. More expressive and sophisticated formalisms have been developed (first and second order extensions of well known formalisms as well as new multi-modal and multi-agent approaches). Finally the consideration of some paradoxes concerning the relation of probability and qualitative belief, as well as problems related to judgment aggregation, have motivated researchers to rethink the very foundations of the Kripkean semantic program in modal logic. We survey some of these new developments as well as the philosophical problems that motivated them.

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Proof theory and meaning: three case studies

Greg Restall

Many of us are attracted to the idea that inference rules play a role in determining meanings. In this talk I will illustrate options for infererentialist approaches to meaning by looking at three case case studies discussed in the recent literature: (1) classical logic, (2) modalities, and (3) inductive and coinductive types.

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Interpretations in philosophical logic

Albert Visser

In my talk, I will discuss the use of interpretations as a tool for comparing theories. The talk consists of two parts. In the first half I will discuss some general issues.

- I will sketch the various notions of interpretation and the various notions of sameness of theories definable in terms of interpretations.
- I will compare comparison of theories based on interpretations with other notions of comparison, like conservativity.
- I will comment on the question: what does it tell us when we have an interpretation of one theory in another?

In the second half, I will illustrate a use of interpretations to study the Predicative Frege Hierarchy obtained by iterating Predicative Comprehension plus Frege Function. (See for a description John Burgess' book Fixing Frege.) Roughly, the result is that, modulo mutual interpretability, we get Robinson's Arithmetic Q plus all iterated consistency statements for Q, so $Q + \mathrm{con}(Q) + \mathrm{con}(Q + \mathrm{con}(Q)) + \ldots$. The second level of this hierarchy, i.e., $Q + \mathrm{con}(Q)$, is mutually interpretable with Elementary Arithmetic. It is open whether there is a natural hierarchy of faster and faster functions corresponding to the iterated consistencies.

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Special Session:

Proof Complexity and Nonclassical Logics

Proof systems for modal logics

Emil Jeřábek

We discuss some issues in proof complexity of calculi for propositional modal and intermediate logics. We consider the usual Frege ("Hilbert style") proof systems, as well as their variants (extended Frege, substitution Frege) inspired by the classical case. We are interested in polynomial simulations, lower bounds, feasible disjunction properties, feasible partial conservativity results, and similar questions.

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Substructural fuzzy logics

George Metcalfe

In this talk I will discuss a family of substructural logics defined by algebras based on the real unit interval [0,1]; well known examples being Łukasiewicz logic and Gödel–Dummett logic. Gentzen systems can often be obtained for these logics simply by extending sequent calculi for substructural logics like Linear Logic to the level of hypersequents. Moreover, the elimination of a special "density rule" from proofs in such calculi can be used to show completeness results for both Gentzen and Hilbert systems for the logics. Indeed, in certain cases, conditions that guarantee cut-elimination for the calculi also guarantee density-elimination, providing a syntactic characterization of the logics from this family.

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Complexity problems for substructural logics

Alasdair Urquhart

Substructural logics such as relevance logics and linear logics are the most complex nonclassical propositional calculuses investigated to date. They include undecidable systems, as well as logics that are decidable, but provably intractable. This talk surveys some of the work in the area, and also lists some open questions.

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Special Session:

Set Theory

Partitioning κ-fold covers into κ many subcovers

Márton Elekes

Motivated by a question of A. Hajnal we investigate the following set of problems. Let X be a set, κ a cardinal number, and $\mathbf{H}$ a family that covers each x in X at least κ times. Under what assumptions can we decompose H into κ many subcovers? Equivalently, under what assumptions can we colour H by κ many colours so that for each x in X and each colour c there exists H in $\mathbf{H}$ of colour c containing x?

The assumptions we make can be, e.g., that $\mathbf{H}$ consists of open, closed, compact, convex sets, or polytopes in $\mathbb{R}^n$, or intervals in a linearly ordered set, or we can make various restrictions on the cardinality of X, $\mathbf{H}$, or elements of $\mathbf{H}$.

Besides numerous positive and negative results, many questions turn out to be independent of the axioms of set theory.

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This is a joint work with T. Mátrai and L. Soukup.

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Maximality principles for closed forcings

Gunter Fuchs

I am going to talk about the modifications of the maximality principles, which were originally introduced by Joel Hamkins, that result from restricting them to subclasses of $< \kappa$-closed forcings. The subclasses I consider are the entire class of $< \kappa$-closed forcings, the $< \kappa$-directed closed forcings and the collapses to κ. So for example, the maximality principle for $< \kappa$-closed forcings says that any statement that can be forced to be true by a $< \kappa$-closed poset in such a way that it remains true in any further generic extension by $< \kappa$-closed forcing, is already true. The talk will center around the following aspects: Consistency of the principles, the compatibility with large cardinal properties of κ, the relationships between the different versions of the principle, outright consequences, the possibilities of combining them in the sense that they hold at many regular κ at the same time, and the limitations of the extent to which they may be combined.

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Scott's problem for proper Scott sets

Victoria Gitman

Given a model of Peano Arithmetic (PA), define its "standard system" to be the collection of subsets of the natural numbers that arise as intersections of the definable sets of the model with its standard part. In 1962, Scott observed that standard systems satisfy certain basic computable theoretic and set theoretic properties. A collection of subsets of the natural numbers having these properties came to be known as a "Scott set". As a partial converse, Scott showed that countable Scott sets are exactly the "countable" standard systems and in 1982, Knight and Nadel extended his result to standard systems of size ω_1. The question of whether Scott sets are exactly the standard systems of models of PA came to be known as "Scott's problem". I will introduce the history of Scott's Problem and talk about my results for standard systems of size ω_2, which were obtained using forcing constructions with models of PA together with the Proper Forcing Axiom. In particular, I will define the notion of a proper Scott set and show under PFA that every proper arithmetically closed Scott is the standard system of a model of PA. I will also focus on the set theoretic questions (and answers) involving the existence of proper Scott sets.

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The Urysohn sphere is oscillation stable

Lionel Nguyen Van Thé

In 1994, Odell and Schlumprecht built a uniformly continuous map from the unit sphere of the Hilbert space into the unit interval and which does not stabilize on any isometric copy of the sphere. This result allowed to show that the Hilbert space has a property known as 'distortion'. The purpose of the present talk is to show that this situation is different when working with another remarkable metric space sharing many common features with the Hilbert sphere: the Urysohn sphere.

This is joint work with Jordi López Abad.

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Classifying measure preserving actions up to conjugacy and orbit equivalence

Asger Törnquist

Extending a well-known theorem by Foreman and Weiss we show: If H is an infinite subgroup of a discrete countable group G, then the measure preserving actions of H that can be extended to G cannot be classified by countable structures. As a consequence, the measure preserving actions of a countable group with the relative property (T) cannot be classified up to orbit equivalence by countable structures.

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The constructible universe for the anti-foundation axiom system ZFA

Matteo Viale

Aczel [1], and independently Forti and Honsel [2], introduced a strengthening of the axiom of foundation asserting the existence of a unique transitive collapse for every binary relation which is a set. The first order axiomatization of set theory with foundation replaced by this axiom is currently named in the literature ZFA. Forti and Honsel [3] also show that if M and N are transitive models of ZFA such that $M \cap WF = N \cap WF$, then $M = N$ (where WF is the definable class of well-founded sets). We exploit this idea to show that there are natural Gödel operations which are absolute for the theory ZFA such that the smallest transitive class which is closed under these Gödel operation is the "constructible" universe of the theory ZFA. The results that we present appeared in [4].

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Contributed Talks

Completion of numberings

Serikzhan A. Badaev, Sergey S. Goncharov, and Andrea Sorbi

Complete numberings play an important role in computability theory: they have an effective fixed point property, and their degrees are not splittable, while the completion operator gives us a regular way to construct a complete numbering α_a for any numbering $\alpha\colon \omega \longrightarrow \mathcal{A}$ and any special element $a \in \mathcal{A}$. We continue the study of the completion operator, started in [1].

As observed by Ershov, [2], $\alpha \leq \alpha_a$, and $\alpha_a \leq \beta$ for every numbering $\beta \geq \alpha$ which is complete with respect to a. In particular, $\alpha_a \equiv \alpha$ if and only if α is complete with respect to a.

We prove that no minimal numbering can be complete and, for every Friedberg numbering α, the interval $(\deg(\alpha), \deg(\alpha_a))$ consists of incomplete numberings (relative to any element of $\mathcal{A}$).

We show that, for some numbering α of a two-element set $\mathcal{A}$, the segment $[\deg(\alpha), \deg(\alpha_a)]$ is isomorphic to the upper semilattice of c.e. **m**-degrees. This is a partial solution to an open problem posed in [1].

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The algebraic structure of quasi-degrees

Ilnur Batyrshin

Tennenbaum (see [5]) introduced the notion of quasi-reducibility for sets: $A \leq_Q B$ if there is a computable function f such that $x \in A \Leftrightarrow W_{f(x)} \subseteq B$. Q-reducibility is a natural generalization of many-one reducibility (m-reducibility) and is closely connected with enumeration reducibility. Besides, on c.e. sets Q-reducibility implies T-reducibility: if a c.e. set $W \leq_Q A$ then $W \leq_T A$.

Quasi-reducibility plays [4] the main part in the Marchenkov's solution to the Post's problem in his spirit. He showed that no $\eta-$hyperhypersimple set is Q-complete and since Q-reducibility coincides with T-reducibility on semirecursive c.e. sets, each semirecursive noncomputable η-hyperhypersimple set is a solution to the Post's problem.

It also turned out that quasi-reducibility is closely connected to the word problems of groups: Dobritsa proved that for every set of natural numbers X there is a word problem with the same Quasi-degree as that of X, and Belegradek [2] showed that for any computably presented groups G and H, if G is a subgroup of every algebraically closed group of which H is a subgroup, then G's word problem must be quasi-reducible to that of H.

The study of the algebraic structure of Quasi-degrees of c.e. sets was started by the paper of Downey, LaForte and Nies [3], who established the density of the c.e. Q-degrees. Arslanov and Omanadze [1] started the study of the algebraic structure of Quasi-degrees of n-c.e. sets. In my report I will present some new results in this direction.

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Multiplicative quantifiers in fuzzy and substructural logics

Libor Behounek and Petr Cintula

In substructural logics without contraction or weakening there are two sets of connectives: lattice-theoretical ("additive") and group-theoretical ("multiplicative"). There is a long-standing problem of extending this distinction to quantifiers in first-order substructural logics. While the additive quantifiers can easily be defined (as the infimum or supremum in the lattice of truth values) and axiomatized (by Rasiowa's [1] axioms for quantifiers in implicative logics), the same cannot easily be done for multiplicative quantifiers.

Deductive fuzzy logics can be viewed as substructural logics that enjoy the prelinearity property [2, 3]. We show how multiplicative quantifiers of various strength can be introduced in first-order fuzzy logic. Moreover we demonstrate that the additive quantifiers suffice for the development of sufficiently rich higher-order fuzzy logic [4], in which the multiplicative quantifiers become definable, and an internal theory of generalized quantifiers can be developed in its framework. The latter method can be extended to a broader class of substructural logics; the problem of multiplicative quantifiers can thus be bypassed through a detour to higher-order substructural logics based on additive quantifiers only.

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Interpretability in PRA

Marta Bilkova, Dick de Jongh and Joost J. Joosten

In this paper we study **IL**(PRA), the interpretability logic of PRA. As PRA is neither an essentially reflexive theory nor finitely axiomatizable, none of the known arithmetically complete interpretability logics applies to PRA. In this paper, we take two arithmetical properties of PRA and see what their consequences on the modal logic **IL**(PRA) are. These properties are reflected in Beklemishev's Principle B, and Zambella's Principle Z.

It is possible to define a class of modal formulae which are under any arithmetical realization provably Σ_2 in PRA, so-called *essentially* Σ_2*-formulas*, we write ES_2 ($\mathcal{A}$ stands for the set of all modal interpretability formulae):

$$\begin{array}{lcl}\mathsf{ED}_2 & := & \Box\mathcal{A} \mid \neg\mathsf{ED}_2 \mid \mathsf{ED}_2 \wedge \mathsf{ED}_2 \mid \mathsf{ED}_2 \vee \mathsf{ED}_2 \\ \mathsf{ES}_2 & := & \Box\mathcal{A} \mid \neg\Box\mathcal{A} \mid \mathsf{ES}_2 \wedge \mathsf{ES}_2 \mid \mathsf{ES}_2 \vee \mathsf{ES}_2 \mid \neg(\mathsf{ES}_2 \rhd \mathcal{A})\end{array}$$

We can now formulate Beklemishev's principle B:

$$B := A \rhd B \to A \wedge \Box C \rhd B \wedge \Box C \qquad \text{for } A \in \mathsf{ES}_2,$$

and Zambella's principle Z:

$$Z \quad (A \equiv B) \to A \rhd A \wedge B \quad \text{for } A \text{ and } B \text{ in } \mathsf{ED}_2.$$

We prove principle B to be arithmetically sound. Next, both principles and their interrelation are submitted to a modal study. In particular, we prove a frame condition for B. The frame condition is either infinite first order, or contains a quantification over some sort of bi-simulation. Moreover, we prove that Z follows, both semantically and syntactically from a very restricted form of B.

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Simple monadic theories

Achim Blumensath

We give an overview over recent developments concerning the model theory of monadic second-order logic. On the one hand, there are tree-like structures whose monadic theory is simple enough to develop a structure theory. On the other hand, there are structures with definable pairing functions where monadic second-order logic is as expressive as full second-order logic. According to a conjecture of Seese [2] these two cases form a dichotomy: either a structure is 'treelike' or it has a definable pairing functions. For graphs (or structures with relations of arity at most 2) a variant of this conjecture has recently been proved by Courcelle and Oum [1]. In this talk we will present first partial results concerning the general case. We consider structures without pairing function and we prove that the partition width of such structures is bounded by $2^{2^{\aleph_0}}$.

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Definable well-ordering, the GCH, and large cardinals

Andrew Brooke-Taylor

It is known that one may force a definable well-order of the universe to exist, for example by McAloon's technique of coding into the continuum function, or using Jensen coding to make the universe constructible from a real. However, it would be nice to be able to force to obtain a definably well-orderable universe where the GCH holds and very large cardinals exist, a situation that cannot be achieved by either of those methods. We show how one can achieve it, using a class length forcing iteration which may be constructed so as to preserve a proper class of such large cardinals as n-superstrong, n-huge, or κ^+-supercompact κ.

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Computational Complexity of NL1 with Assumptions

Maria Bulinska

We take into consideration Non-associative Lambek Calculus with identity (NL1) enriched with a finite set of arbitrary assumptions and some of extensions of this system such as NL1 with permutation and Generalized Lambek Calculus (i.e. the system with n-ary operations) with identities. De Groote and Lamarche in [2] established the polynomial time decidability for Classical Non-associative Lambek Calculus. Buszkowski in [1] showed that systems of Non-associative Lambek Calculus with assumptions are also decidable in polynomial time and generate context-free languages. The same holds for systems with unary modalities, studied in Moortgat [5], n-ary operations, and the rule of permutation, studied in Jäger [3]. In order to obtain the P-TIME decision procedure for NL1 with the finite set of nonlogical axioms we adapt the method used by Buszkowski [1]. This method does not rely on cut elimination which is not available for systems with additional assumptions. Then, using the results for NL1 we prove that considered extensions are decidable in polynomial time.

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Admissible rules for the multimodal logic of knowledge and time LTK_1

Erica Calardo

Multi-modal propositional logics with operators representing both time and knowledge are particularly effective to describe the interaction of agents throughout the flow of time ([2, 1, 3, 4, 5, 7]). These systems are usually generated by adding to an existing propositional logic two sets of modalities: one to model the flow of time and one to describe agents' knowledge. The interaction of such modalities gives a precise account of the dynamic development of agents' knowledge. However, despite the power of multi-modal propositional logics, multi-modal languages can only express formulae which are static in a way: the statements only fix a fact, and cannot handle a changing environment, although this is required to model human reasoning, computation and multi-agent environments. Sometimes it might be more useful to discover what follows given some premises, rather than knowing logical truths. For this purpose, inference rules, or logical consecutions, are a core instrument.

Our research aims at investigating a multi-modal propositional logic, LTK (Linear Time and Knowledge), which combines tense and knowledge modalities. This logic is semantically defined as the set of all LTK-valid formulae, where LTK-frames are multi-modal Kripke-frames combining a linear and discrete representation of the flow of time with special S5-like modalities, defined at each time cluster and representing agents' knowledge.

So far we have proved that: (i) LTK has the finite model property [2]; (ii) LTK has a finite axiomatisation [3]; (iii) LTK_1, a weaker version of LTK with only one agent operating in the system, is decidable with respect to its admissible inference rules [1]. Our latest results is to show that LTK_1 has a finite basis for admissible inference rules, i.e., all those rules under which the logic itself is closed.

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A forcing extension of a $(\omega_2, 1)$ simplified morasses with no $(\omega_2, 1)$ simplified morasses with linear limits using an unfoldable cardinal

Franqui Cárdenas

Lee Stanley [3] has observed that if κ is supercompact cardinal then there is a forcing extension where there is a $(\omega_2, 1)$-simplified morass but there is no $(\omega_2, 1)$-simplified morass with linear limits. Using the lottery preparation, a technic developed by Joel Hamkins in [2], we just need a strongly unfoldable cardinal for the same observation.

Let κ an unfoldable cardinal, the forcing $\mathbb{P}$ which adds a $(\kappa, 1)$-simplified morass is κ-closed and it is also κ^+-c.c., although we can't extend the unfoldable embeddings in the ground model using directly this forcing (since it is too big to be in any M transitive model of ZFC^- of size κ), it is possible to do it thanks to a properness condition [1].

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Strong jump-traceability I: the computably enumerable case

Peter Cholak, Rod Downey, and Noam Greenberg

This talk will discuss a project which determines the relation between Turing degrees which are *super jump traceable* and those which are *K-trivial*. By work of Nies and others the K-trivials are very robust class of degrees; for example, see [2].

We say that a function $h\colon \omega \to \omega \setminus \{0\}$ is an *order* (Schnorr) if h is computable, nondecreasing and $\lim_s h(s) = \infty$. We say that a function $f\colon \omega \to \omega$ is *computably traceable* with respect to the order h if there is a computable sequence $\langle F_x \rangle_{x<\omega}$ of finite sets such that for all x, $|F_x| \leq h(x)$ and $f(x) \in F_x$. We will say that a degree $\mathbf{a}$ is computably traceable iff there is some order h such that every f of degree $\mathbf{a}$ or less can be computably traced with respect to h. Finally, we will say that $\mathbf{a}$ is *strongly* computably traceable iff it is computably traceable with respect to any order. Here the idea is that the real is *computationally feeble*, in the sense that we have very good approximations to computations using A as an oracle. Perhaps one would expect that such reals would be highly non-random.

We have shown:

Theorem 1 ([1]). *Every c.e. strongly jump-traceable set is K-trivial.*

Thus for the first time, we have an example of a combinatorial property that at least *implies* K-triviality. The proof of this result relies on a new combinatorial technique using a kind of amplification of the traceability along the lines of the decanter or golden run method. It is beyond known technology; we believe that it could have other applications within computability theory and randomness.

On the other hand we also prove the following.

Theorem 2 ([1]). *There is a K-trivial c.e. set that is* not *strongly jump-traceable. Indeed it is not jump traceable with a bound of size roughly* $\log \log n$.

This is the first example of a class defined by cost functions which we know does not coincide with the K-trivials in the proof technique is novel, since it is the first time a cost function has been used which still allows for the defeat of one involving Kolmogorov complexity.

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Structural completeness for fuzzy logics

Petr Cintula and George Metcalfe

A consequence relation is structurally complete if and only if all of its proper extensions have new theorems. That is, if a schematic rule is admissible (preserves the set of theorems), then it is derivable in any formal system axiomatizing the consequence relation. Classical logic has this property, but for many families of non-classical logics such as Intermediate, Modal, and Substructural logics, it is relatively rare, often existing only for certain fragments of the systems in question.

In this work we investigate structural completeness for the class of (t-norm based) fuzzy logics. These include Gödel–Dummett logic, known to be structurally complete, and Łukasiewicz logic, where the positive fragment is known to have the property, but not the full logic. We show that all fragments of the third fundamental fuzzy logic, Product logic, are structurally complete, a result that extends to related logics such as Cancellative Hoop Logic and Abelian Logic. We also give very general methods to show that a wide range of logics (fuzzy or otherwise) are not structurally complete. By combining these approaches we are able to answer the question of structural completeness for most (if not quite all) prominent fuzzy logics.

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Changing u_2 by stationary set preserving forcing

Benjamin Claverie

We show that if I is a precipitous ideal on ω_1 and $\theta > \omega_1$ is regular, then there is a stationary set preserving forcing which adds a countable structure (M, J) which iterates to (H_θ, I) in ω_1 steps. Therefore, if there is a precipitous ideal on ω_1 and the universe is closed under sharps, then there is a stationary set preserving forcing which increases u_2. If BMM holds and there is a precipitous ideal on ω_1, then $u_2 = \omega_2$.

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Equiconsistency of choiceless higher Chang conjectures with one Erdős cardinal

Ioanna Matilde Dimitriou and Peter Koepke

We are interested in the connection between cardinals defined by certain combinatorial properties and cardinals satisfying certain model theoretic relations. In particular we will look at Erdős cardinals and generalised Chang conjectures.

Even though with the axiom of choice (AC) several versions of the Chang conjecture have consistency strengths in the realm of strong cardinals, higher or are even inconsistent, without AC we show that some versions of the Chang conjecture are equiconsistent with just one Erdős cardinal.

By a Chang conjecture we mean that for some cardinals κ, κ', λ and λ', the statement $\langle \kappa, \lambda \rangle \to \langle \kappa', \lambda' \rangle$ holds, i.e., for any structure with domain A of size κ with a unary predicate $B \subseteq A$ of size λ in a countable language, there is an elementary substructure $\langle A', B', \dots \rangle \prec \langle A, B, \dots \rangle$ with $|A'| = \kappa'$ and $|B'| = \lambda'$. On the other hand, a cardinal κ is λ-Erdős if it is the least such that $\kappa \to (\lambda)^{<\omega}$ holds, i.e., for every partition f of the finite subsets of κ into two colours, there is a homogeneous set for f of size λ.

We show that for λ a regular cardinal, the existence of a λ-Erdős cardinal κ is equiconsistent with $\langle \lambda^+, \lambda \rangle \to \langle \lambda, \nu \rangle$ for any infinite $\nu < \lambda$. For the forward direction we construct a symmetric model where κ is λ^+ and still λ-Erdős. In that model, $\langle \lambda^+, \lambda \rangle \to \langle \lambda, \nu \rangle$ holds for all ν with $\lambda > \nu \geq \omega$. For the converse we use Jensen's indiscernibles lemma to show that if one of these versions of the Chang conjecture holds, then $(\lambda^+)^V$ is λ-Erdős in K, the Dodd–Jensen core model.

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On the consistency strength of the tree property at the double successor of a measurable cardinal

Natasha Dobrinen and Sy-David Friedman

Given any regular cardinal κ and a weak compact λ above κ, Mitchell showed that λ can be collapsed to κ^+ in such a way that in the forcing extension, κ^+ has the tree property. His methods used a mixed-support iteration of various Cohen forcings. We show that iterated generalized Sacks forcing can achieve the same goal. We then use these iterations to obtain a model where the tree property holds at the double successor of a measurable cardinal, using much weaker hypotheses than were previously known to suffice.

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Logical Inferences and Visualization

Horacio Faas

As inferred from the number of publications and academic meetings on the subject, interest on visualization is greatly renewed nowadays in mathematical and logic circles. Although visualization is usually understood in different ways it is surely closely related with intuition, and it is well known that in the beginnings of past century this one suffered rough attacks as misguiding and not trustworthy. From then on, the reliable inferences were solely those formalized in a language. But even after it was widely accepted the position against intuition there were several opinions remarking its enormous contribution to advances in precise knowledge. ("Einstein thought in images", said Hadamard). It cannot be refused that in daily life we make inferences without appealing to linguistic intermediation. Even more, it seems that there exists some type of inferences made by animal species other than human. As said above, visualization is coming back accompanied sometimes by rigorous presentations to arrive at, in some cases, to accept it like genuine part of demonstrations. In this paper I show a few different approaches to non linguistic inferences, aiming at highlighting the possibility of including diagrammatic and heterogeneous reasoning in proofs. Examples of it should be how to justify newo geometrical knowledge based on pictures (perhaps a "kantian" synthetic a priori knowledge, to be criticized), and the way inferences understood as information extraction by means of information flux presentation would be useful in proofs. I also make some comments about a bit of the history of calculus on this subject, going from Leibnizian diagrams to Cauchy's proof of the fundamental theorem.

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Δ^0_n-categorical linear orderings

Andrey N. Frolov

I'll talk about Δ^0_n-categorical linear orderings. In particular, I will show that there is a strictly Δ^0_{n+1}-categorical linear ordering. A linear ordering is strictly Δ^0_{n+1}-categorical, if it is Δ^0_{n+1}-categorical and it is not Δ^0_n-categorical. Also I'll talk about Δ^0_n-stable linear orderings.

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Computable Models Spectra of Ehrenfeucht Theories

Alexander N. Gavryushkin

This study is devoted to the class of Ehrenfeucht theories, this class of theories has been well studied and has attracted considerable attention.

Definition 1. *A theory is an* ***Ehrenfeucht theory*** *if it has finitely (greater then one) many models.*

Vaught proved that no complete theory has exactly two models. On the other hand, for each $n \geq 3$ there exists a theory with exactly n models. For example, the theory of the model $< Q; \leq; c_0, c_1, \ldots >$, where $< Q; \leq >$ is the natural ordering of rationals and $c_0 < c_1 < \ldots$, has exactly three models. This example can be generalized to give theories with exactly $n \geq 4$ models. Inspired by

Theorem 2. *[1, 4] Let T be an uncountably categorical theory. Then T is decidable if and only if T has a decidable model if and only if all models of T have decidable presentations.*

Nerode posed the following question: If an Ehrenfeucht theory T is decidable then do all models of T have strong constructivizations? It turns out that models of decidable Ehrenfeucht theories are not as well behaved as decidable uncountably or decidable countably categorical theories. For instance, the following theorem is true.

Theorem 3. *[8] For each $n \geq 3$ there exists a decidable Ehrenfeucht theory T_0 that admits elimination of quantifiers, has exactly n models and exactly one model with a strong constructivization.*

In relation to this theorem we make the following comments. First of all we note that the prime model of any decidable Ehrenfeucht theory must have a strong constructivization. This follows from an effective version of the Omitting Types Theorem for decidable theories [7] which is not discussed in this paper. Hence the strongly constructive model of the theory T_0 in the theorem above is a prime model. Secondly, the reason that not all models of T_0 have strong constructivizations is that T_0 has a noncomputable type. Based on this Morley asked the following question that has become known as Morley's problem:

Question 4. *If all types of an Ehrenfeucht theory T are computable then do all models of T have strong constructivizations?*

This is an open problem which has been attempted by many with no success. Ash and Millar obtained several interesting results in the study of this question. One of the results is the following.

Definition 5. *An Ehrenfeucht theory T is* ***persistently Ehrenfeucht*** *if any complete extension of T with finitely many new constants is also an Ehrenfeucht theory.*

Theorem 6. *[1] If T is persistently Ehrenfeucht all of whose types are arithmetical then all models of T have arithmetic presentations.*

In relation to this theorem and Morley's problem, it is interesting to note that the following question, asked by Goncharov and Millar, is still open:

Question 7. *If T is an arithmetic Ehrenfeucht theory whose types are arithmetical, do then all models of T have arithmetic presentations?*

We now briefly discuss the problem of existence of constructive models of Ehrenfeucht theories. As for categorical theories, there has not been much research about finding constructive models for (undecidable) Ehrenfeucht theories. We note that the results in finding constructive models for undecidable Ehrenfeucht theories can be quite different from those about decidable Ehrenfeucht theories. We give an example. If all types of an Ehrenfeucht theory T are computable then T must have at least three strongly constructive models (a proof of this can, for example, be found in [5]). Therefore for any decidable Ehrenfeucht theory T with exactly three models, the saturated model of T has a strong constructivization if and only if all models of T have strong constructivizations. We also recall that the prime model of every decidable Ehrenfeucht theory has a strong constructivization. In contrast to this, in [6] the following theorem is proved:

THEOREM 8. *There exists an Ehrenfeucht theory with exactly three models of which only the saturated one has a constructivization.*

We conclude with the following research proposal.

PROBLEM 9. *Work towards characterizing strongly constructive or/and constructive models of Ehrenfeucht theories.*

It was a citation from [2]. And here there is one step on the path to the solution of 9.

THEOREM 10. *There exists a theory T such that*

1. *T is Ehrenfeucht theory;*
2. *The prime model of T has computable presentation;*
3. *The saturated model of T has computable presentation;*
4. *There exists a model of T with no computable presentation.*

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Towards a notion of p-adic closure for a ring

Nicolas Guzy

In [1] L. Bélair generalizes results in [4] by giving an axiomatization, denoted by $ALpC_{e,f}$, of local p-adically closed rings of p-rank d, i.e., henselian local rings with residue field which are p-adically closed field of p-rank d ($d = ef$ with e and f fixed). The results extend the ones obtained by E. Robinson about the p-adic spectrum. Then he completes the theory $ALpC_{e,f}$ in a scheme of axioms $AIpC_{e,f}$ whose models are henselian valuation rings whose residue fields are p-adically closed fields of p-rank d and value groups are divisible.

Particular examples of models of $AIpC_{e,f}$ are provided and studied in Section 3 of [1]: if C is an affine curve on $\mathbb{Q}_p$ then L. Bélair shows that for any prime non maximal ideal $\mathfrak{p}$ of the ring of continuous definable functions $C \to \mathbb{Q}_p$, denoted $\mathcal{C}(C)$, we have $\mathcal{C}(C)/_{\mathfrak{p}}$ is a model of $AIpC_{e,f}$.

To this effect, the powerful tool used in this paper is the existence of an homeomorphism between the p-adic spectrum of $\mathbb{Q}_p[V]$ (V is an affine variety over $\mathbb{Q}_p$) and the ring spectrum of $\mathcal{C}(V)$.

In [2], L. Bélair generalizes his previous approach to the ring of continuous definable functions on V over $\mathbb{Q}_p$, denoted by $\mathcal{C}(V)$, for any affine p-adic variety V. He studies, in particular, first-order properties of the quotient rings $\mathcal{C}(V)/\mathfrak{p}$ by a prime ideal $\mathfrak{p}$ of $\mathcal{C}(V)$.

This talk is devoted to give an account of the work in [3]. In this work, we introduce a new class of rings called p-adically closed rings. For this purpose, we build the p-adic closure of a given ring A in a similar way as in [5] for the real closure of a ring. Namely, the topological properties of the p-adic spectrum of A plays a central role in this construction.

Then it allows us to define the notion of a p-adically closed ring and to generalize the results from [1] and [2] to the p-adically closed rings.

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Complexity of fuzzy predicate logics with witnessed semantics

Petr Hájek

Investigated are continuous t-norm based fuzzy predicate logics with standard semantics (the set of truth degrees is the closed real unit interval), as introduced in my Metamathematics of fuzzy logic (Kluwer 1998); in particular, Łukasiewicz, Gödel and product logic. Semantics is Tarski-style; over a crisp non-empty domain predicates are interpreted by fuzzy ($[0,1]$-valued) relations, the value of a universally quantified formula being the infimum of the values of instances and analogously for existential quantification and supremum. A tautology is a formula having the value 1 in each interpretation; a formula is satisfiable if it has the value 1 in some interpretation. The arithmetical complexity of the set of tautologies and the set of satisfiable formulas is known to be: for Łukasiewicz Π_2-complete, Π_1-complete; for Gödel Σ_1-complete, Π_1-complete; for product logic non-arithmetical, non arithmetical. An interpretation is witnessed if the truth value of each universally quantified formula is the minimum of truth values of instances (the infimum is taken) and the truth value of each existentially quantified formula is the maximum of values of instances. A witnessed tautology is a formula having the value 1 in each witnessed interpretation; similarly for witnessed satisfiability. Now the complexity of the set of witnessed tautologies/witnessed satisfiables: for Łukasiewicz Π_2-complete, Π_1-complete; for Gödel Σ_1-complete, Π_1-complete (for both logics the same complexity as without the assumption of witnessedness); for product logic the set of witnessed tautologies is Π_2-hard (nothing more is known) whereas the set of witnessedly satisfiable formulas is Π_1-complete. For Göodel and product the set of witnessedly satisfiable formulas is equal to the set of formulas in the classical Boolean logic. Much more is known; a survey will be given.

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Motivic measure for pseudo-finite like fields

Immanuel Halupczok

Let T be the theory of pseudo-finite fields. To understand the definable sets $\mathrm{Def}(T)$, it is helpful to have some invariants, i.e. maps from the definable sets to somewhere else which are invariant under definable bijections. In [1], Denef and Loeser constructed a very strong such invariant: to each definable set $X \in \mathrm{Def}(T)$, they associated a virtual motive $\mu(X)$. In this way one gets all the cohomological invariants (like the Euler characteristic or the Hodge polynomial) for arbitrary definable sets of T.

An interesting question is whether this generalizes to other fields as well. Indeed, suppose now that G is any pro-cyclic torsion-free group and T is the theory of perfect, pseudo-algebraically closed fields of characteristic zero with absolute Galois group G. (For $G = \hat{\mathbb{Z}}$, these are just the pseudo-finite fields of characteristic zero.) We will generalize the construction of Denef-Loeser to this situation. In addition, even in the pseudo-finite case we will find other invariants which yield additional information, not yet contained in the invariant μ of Denef-Loeser.

The main idea is the following: Let $G \subset G'$ be two groups as above and let T and T' be the corresponding theories. Then we will construct a map $\theta\colon \mathrm{Def}(T) \to \mathrm{Def}(T')$. Taking $G' = \hat{\mathbb{Z}}$ then yields a map $\mu \circ \theta$ from the definable sets of T to the virtual motives. Identifying G with one of its subgroups yields interesting maps from $\mathrm{Def}(T)$ to itself.

Details can be found in [2].

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On uncountably categorical approximations and Gromov-Hausdorff limits

Assaf Hasson and Boris Zilber

In recent years Zilber and others found intriguing relations between Hrushovski's free amalgamation constructions and conjectures in transcendental number theory (variants on Schanuel's conjecture) and Diophantine geometry (variants on the Conjecture on the Intersection with Tori). However, the other part of the construction, the collapse, though - model theoretically - it can be interpreted as a generalization of the notion of smooth approximation, is still missing an extra-logical interpretation. In the talk I will present recent attempts to introduce a notion of metric approximations of structures, and their connection to both Hrushovski's constructions and to ideas borrowed from mathematical physics.

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Higher-order reverse topology

James Hunter

Reverse Mathematics studies equivalences between logical axioms and mathematical theorems. Traditional Reverse Mathematics is limited to subsystems of second-order arithmetic, and thus can consider only those mathematical theorems expressible in the language of second-order arithmetic.

In a recent paper, Kohlenbach showed that Reverse Mathematics extends nicely to higher-order theories, and examined certain higher-order statements in mathematical analysis. The same techniques can be applied to statements about topological spaces of size continuum, allowing for an examination of statements more general than those studied by traditional Reverse Mathematics.

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A remark on a characterization of non-forking in generic structures

Koichiro Ikeda

Let L be a countable relational language and $\mathbf{K}$ a class of finite L-structures with non-negative predimension. Then a countable L-structure M is said to be $\mathbf{K}$-generic, if (i) any finite $A \subset M$ belongs to $\mathbf{K}$; (ii) for any finite $A \leq B \in \mathbf{K}$ with $A \leq M$ there is a copy B' of B over A with $B' \leq M$; (iii) there are finite $A_0 \leq A_1 \leq A_2 \leq \cdots \leq M$ with $M = \bigcup_i A_i$. Let M be a saturated generic structure and $\mathcal{M}$ a big model of $\mathrm{Th}(M)$. In [1], [2] and [3], non-forking in $\mathcal{M}$ has been characterized as follows: For any $A \leq B, C \leq \mathcal{M}$ with $A = B \cap C$ *algebraically closed*, $\mathrm{tp}(B/C)$ does not fork over A if and only if B, C are free over A and $BC \leq \mathcal{M}$. It is seen that one cannot remove the condition that A is algebraically closed in the above statement. Our aim is to generalize the statement as follows: For any $A \leq B, C \leq \mathcal{M}$ with $A = B \cap C$, $\mathrm{tp}(B/C)$ does not fork over A if and only if B, C are free over A and $BC \cup \mathrm{acl}(A) \leq \mathcal{M}$. As a corollary, it can be proved that there is no saturated generic structure that is superstable but not ω-stable.

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What if computers could count to infinity?

Chris Impens and Sam Sanders

This paper describes work in progress in weak systems of arithmetic and applications. We define a nonstandard version of the theory of polynomial time functions in which the usual fundamental principles of nonstandard mathematics, like saturation and transfer, are integrated. Our main result is that, over the finite numbers, the nonstandard polynomial time functions contain the recursive functions. Given the big difference in computational complexity between the standard polynomial time functions and the recursive ones, the result is surprising.

Hypercomputation with computers effectively executing infinitely many calculations in a finite time has already been considered in quantum mechanics and elsewhere. Our nonstandard theory of polynomial time functions is a very natural abstraction of this computational concept, and allows unexpected insights in the feasibility and power of such a hypercomputer.

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The second author will present the contributed talk.

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Separating notions of randomness

Bart Kastermans and Steffen Lempp

The intuitive concept of randomness allows several possible concise definitions, many of which have been shown to be inequivalent. We discuss some of these notions and present a new separation result: Not every "injectively partially computably random" (and thus in particular not every "permutation partially computably random" set) is Martin-Löf random. This is a partial result toward separating Martin-Löf randomness from Kolmogorov–Loveland randomness, one of the main open questions in this area. (See J. Miller/Nies, BSL, vol. 12 (2006), pp. 390–410, for more background on this area.)

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Non-splitting enumeration degrees

Thomas F. Kent and Andrea Sorbi

There are not many properties that hold for all proper ideals of Σ^0_2-enumeration degrees. For example, it is known that there are ideals of Σ^0_2-enumeration degrees which contain only properly Σ^0_2 degrees except $\mathbf{0}_e$ (and hence no low-degree), while there are also non-trivial ideals consisting of only low-degrees. The authors have shown that any proper ideal contains a non-splitting degree, and thus the non-splitting degrees are downwards dense in the Σ^0_2-enumeration degrees. In proving this result, they also introduce a new form of permitting, which allows them to build below an arbitrary Σ^0_2-set.

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Pregroups: their logic and application for natural languages

Aleksandra Kiślak-Malinowska

The calculus of pregroups, introduced by J. Lambek in 1999, is an essential strengthening of Lambek syntactic calculus and can be treated as an attractive alternative.
The calculus of pregroups has initially been formulated in the form of a rewriting system. The first axiom system for the calculus of pregroups was proposed by W. Buszkowski in 2001. Here we want to present this calculus in both forms and also provide additional refinements of this system - pregroups with modalities. Attempts to apply this calculus for natural languages have been made among others for English, German, French, Italian, etc. We want to present the application of pregroups for some grammatical structures of Polish language.

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Complexity of isomorphism for countable models of ω-stable theories

Martin Koerwien

We discuss the relationship between the notion of depth of complete first-order ω-stable theories having NDOP (i.e. "classifiable" in the sense of S. Shelah) and the complexity of their classes of countable models in terms of Borel reducibility as introduced by H. Friedman and L. Stanley.

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The polynomial and linear time hierarchies in weak arithmetic

Leszek Aleksander Kołodziejczyk and Neil Thapen

We prove a number of conditional independence results concerning the relationship between the linear and polynomial time hierarchies in PV and S^1_2. Our general assumption is that integer factoring is hard, in the sense that there does not exist a probabilistic polynomial time algorithm for factoring. Under this assumption, we show that there exists a model of PV in which the two hierarchies differ. The proof technique cannot be extended to S^1_2, but can be modified to yield a model of S^1_2 in which NP is not contained in the second level of the linear hierarchy. We then show that there exists a model of S^1_2 in which the hierarchies are equal. As a corollary of the proof, we obtain the existence of a model of S^1_2 in which PH (defined in terms of so-called strict Σ^b_n classes) does not collapse.

Our methods are model-theoretic and rely on the analysis of variants of the weak pigeonhole principle for polynomial time functions. A separate, though similar, model-theoretic argument shows unconditionally that there is a model of the very weak theory V^0 in which the linear and polynomial hierarchies are different.

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On formulas in one variable and logics determined by wheel frames in NEXT(**KTB**)

Zofia Kostrzycka

In this paper we investigate normal modal logics over $\mathbf{T_2} = \mathbf{KTB} \oplus \Box^2 p \to \Box^3 p$ logic. Although the $\mathbf{T_2}$ logic is characterized by the class of reflexive symmetric and two-step transitive frames, there is very few results concerning it.

Yutaka Miyazaki in [2] considered logics determined by the so-called wheel frames. On the base of these frames and by using the splitting technique effectively, he constructed a continuum of normal modal logics over $\mathbf{T_2}$ logic.

In this paper we characterize the wheel frames by formulas written in one variable. On this purpose we take advantage of the infinite sequence of non-equivalent formulas in one variable from [2].

Theorem 1. *There is a continuum of normal modal logics over* $\mathbf{T_2}$ *logic defined by formulas in one variable.*

We also characterize the connection between the notion of diameter of a frame with a tail, and locally tabular members in $NEXT(\mathbf{T_2})$.

Theorem 2. *Let* $L \in NEXT(\mathbf{T_2})$ *logic and* L *is characterized by frames with a tail. Then:*
L *is locally tabular iff* L *has a finite diameter.*

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Bounded arithmetic and LOGCFL

Satoru Kuroda

The class LOGCFL is defined to be the class of sets LOGSPACE-reducible to some context-free language. This class is equivalent to the class SAC^1 of sets which are computed by some semi-unbounded fan-in, logarithmic depth, polynomial size circuit families.

Using the latter characterization, we define a two-sort bounded arithmetic $V\text{–}Q^{SAC}(\Sigma_0^B)$ [2] which has a bit comprehension axiom for formulae stating that there is a tree certificate of SAC^1 circuit on a given input string. Then we prove that a function is provably total in $V\text{–}Q^{SAC}(\sum_0^B)$ if and only if it is polynomially bounded and bitwise computable in LOGCFL.

The main feature of the proof is that $V\text{–}Q^{SAC}(\Sigma_0^B)$ can prove the closure of SAC^1 under complementation. This closure property of SAC^1 was proved by Borodin et al. [1] using the inductive counting argument and our proof is a formalization of it within the system.

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Signed-bit representations of real numbers and the constructive Stone–Yosida theorem

Robert S. Lubarsky and Fred Richman

The signed-bit representation of reals, as developed classically, is like binary, only in addition to 0 and 1 you can also use -1. This representation lends itself especially well to the constructive (intuitionistic) theory of the reals. (For background on constructive analysis, see [1] or [2].) We develop the signed-bit equivalents of three common notions of real numbers: Dedekind cuts, Cauchy sequences, and Cauchy sequences with moduli of convergence.

This theory is then applied to representations of Riesz spaces. A Riesz space is a lattice ordered vector space (here taken to be over the rationals), and a representation of such is a homomorphism into the reals. The canonical example of a Riesz space is a space of real-valued functions, and a representation is evaluation at a point in the domain. Constructively, in order to find a point in the domain, one must often make an additional assumption, such as Dependent Choice. This is proven for example in [3], where the authors ask whether DC is necessary for this result. In this talk, the existence of (appropriate) Riesz space representations is recast in terms of signed-bit representations, and a possible way to show that to be independent from the regular axioms of set theory sans DC, using a topological model (as in [4], [5], and [6]), is then sketched.

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Unary quantifiers and built-in successor

Kerkko Luosto

Unary quantifiers binding a single formula were introduced by Andrzej Mostowski 50 years ago. The notion of generalized quantifiers was subsequently enhanced by Lindström 1966, but unary quantifiers remain an interesting special case. The definability theory of unary quantifiers is well-understood and leads to some interesting combinatorics involving Ramsey theory (see, e.g., Nešetřil and Jouko Väänänen 1996 or Luosto 2000). However, the impact of built-in relations in the structures has not yet been thoroughly explored. There are some indications that the definability theory in ordered structures is difficult (Luosto 2004). Here, we discuss the case of built-in successor relation and show that except for the change in quantifier ranks, built-in successor is not strong enough to enhance the expressive power of unary quantifiers.

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Nice topologies of Polish G-spaces and admissible sets

Barbara Majcher-Iwanow

Let G be a closed subgroup of the group S_∞ of all permutations of ω. Let $(\langle X, \tau\rangle, G)$ be a Polish G-space. H.Becker has defined *nice topologies* of X by the property that they are finer than τ, the G-action remains continuous and there is a countable basis (*nice basis*) consisting of clopen sets which are invariant with respect to some open subgroups of G and satisfy some further natural conditions (see H.Becker, Topics in invariant descriptive set theory, Ann.Pure Appl.Logic, 111(2001), 145 - 184).

For every $x \in X$ we construct an admissible set $\mathbb{A}_x$ and study nice bases which can be coded in $\mathbb{A}_x$ in some canonical way. Analysing definability in $\mathbb{A}_x$ of codes of some natural families of Borel sets and relations between them we can consider complexity of these objects. Moreover it turns out that the condition that x is generic with respect to the nice topology can be expressed in the language of $\mathbb{A}_x$.

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Probabilistic representation of fuzzy logics

Ondrej Majer and Libor Behounek

The idea of probabilistic interpretation of fuzzy logics dates back to the work of Robin Giles, [2, 3]. He showed, using a game-theoretic framework, that a (fuzzy) value of a formula of Łukasiewicz logic can be represented in the terms of probabilities of its subformulas obtained via a Lorenzen-style dialogue game. Recently Christian Fermüller extended Giles' result and proposed a representation of two other principal fuzzy logics - Gödel and Product [1]. The connection between fuzzy logic and probability theory in both Gilles' and Fermüller's work is rather loose - it remains on the level of isolated atomic events which are not assumed to be a part of a single probabilistic space. The main goal of this article is to make the connection more straightforward and to represent a formula of fuzzy logic as a pair of events in a probabilistic space of an appropriate kind. Our second goal is to extend the representation result to a wider class of fuzzy logics - in particular to those obtained as an ordinal sum of the Łukasiewicz, Gödel and Product logics. This gives us a probabilistic representation of an important class of fuzzy logics, namely those which correspond to continuous t-norms (see [4]).

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MLL proof nets as error-correcting codes

Satoshi Matsuoka

The study of the multiplicative fragment of Linear Logic without multiplicative constants (for short MLL) is successful from both semantical and syntactical point of view. In the semantical point of view there are good semantical models including coherent spaces. In the syntactical point of view the theory of MLL proof nets has obtained a firm status without doubt. On the other hand IMLL, an intuitionistic version of MLL is also studied. IMLL can be seen as a subsystem of MLL. IMLL is easier to be studied more deeply than MLL, because we can use intuitions inspired from the conventional lambda-calculus theory as well as graph-theoretical intuitions from the MLL proof nets theory.

In order to study MLL more deeply, how should we do? One approach is to interpret MLL intuitionistically by using Gödel's double negation interpretation. However in such an approach multiplicative constants must be introduced. Definitely introducing multiplicative constants makes things complicated. Another approach we propose in this paper is to adopt *coding theoretic* framework.

Coding theory is very useful for real world applications. A notable example is digital television. Basically, coding theory is to study a way of detecting and/or correcting data that may be true or false. In this paper we propose a novel approach for analyzing proof nets of MLL by coding theory. We define families of proof structures and introduce a metric space for each family. In each family,

1. an MLL proof net is a real code
2. a proof structure that is not an MLL proof net is a false code.

In this talk we describe a summary about results we have obtained so far, using examples.

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There may be infinitely many near coherence classes under $\mathfrak{u} < \mathfrak{d}$

Heike Mildenberger

We show that in the models from [2], where the ultrafilter characteristic is strictly less than the dominating number, there are infinitely many near-coherence classes of ultrafilters. Thus we answer Banakh's and Blass' Question 30 of [1] negatively. By an unpublished result of Canjar, there are at least two classes in these models. On the way we prove some general facts on ultrafilters under $\mathfrak{u} < \mathfrak{d}$. This talk is mainly a report on [3].

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Around splitting and reaping number for partitions of ω

Hiroaki Minami

We investigate the splitting number and the reaping number for partitions of ω. By using c.c.c forcing we show it is consistent with ZFC that add($\mathcal{M}$) is larger than the reaping number for partitions of ω.

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General reducibilities for sets of reals

Luca Motto Ros

Intuitively, a set of reals A is *simpler* than — or as complex as — a set of reals B if the problem of verifying membership in A can be reduced to the problem of verifying membership in B. This observation has led W. Wadge to introduce the notion of *continuous reducibility*, where A is reducible to (i.e. simpler than) B if there is some continuous function $f : \mathbb{R} \to \mathbb{R}$ such that $f^{-1}(B) = A$. The study of this relation (on the Baire space) has led to the development of a very rich and fascinating theory, now known as Wadge theory. We will give a brief history of some of the most important results in this area and present a very general approach to the study of various notions of reducibility on the Baire space.

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Interpretability of the Arithmetic in certain finitely presented groups

Alexey Muranov

A recent article on arXiv.org of Bardakov and Tolstykh shows that the Arithmetic is interpretable with parameters in Thompson's group F. This group F is finitely presented, and is usually regarded as a subgroup of two other groups of Thompson, T and V, which are simple and finitely presented as well. The group V is also a member of a series of simple finitely presented groups described by Higman in 1974. We have extended the result of Bardakov and Tolstykh by interpreting the Arithmetic in all the groups of Thompson and Higman, and have considered some other first-order properties of such groups. In my talk, shall present our results and questions on this subject.

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Positive realizability morphisms and Tarski models

Cyrus Nourani

Definition 1. *A formula is said to be positive if and only if it is built from atomic formulas using only the connectives* $\&, \wedge$ *and the quantifiers* $\forall, \exists$.

Definition 2. *A formula* $\phi(x_1, x_2, \ldots, x_n)$ *is preserved under homomorphisms if and only if for any homomorphisms* f *of a model* A *onto a model* B *and all* $a_1, \ldots, a_n$ *in* A *if* $A \models [a_1, \ldots, a_n]$, *then* $B \models [f(a_1), \ldots, f(a_n)]$.

Theorem 3. *A consistent theory* T *is preserved under homomorphisms if and only if* T *has a set of positive axioms.*

Positive forcing (author 1981) had defined T^* to be T augmented with induction schemas on the Generic diagram functions. That can effectively generates Tarskian models since Tarskian presentations can be created with Skolemization on arbitrary sentences on generic diagrams with the Skolem functions instantiating the generic diagram functions.

Proposition 4. *Let* $\mathcal{R}$ *and* $\mathcal{D}$ *be models for* L. *Then* $\mathcal{R}$ *is isomorphically embedded in* $\mathcal{D}$ *if and only if* $\mathcal{D}$ *can be expanded to a model of the diagram of* $\mathcal{D}$.

Proposition 5. *Let* $\mathcal{R}$ *and* $\mathcal{D}$ *be models for* L. *Then* $\mathcal{R}$ *is homomorphically embedded in* $\mathcal{D}$ *if and only if* $\mathcal{D}$ *can be expanded to a model of the positive diagram of* $\mathcal{D}$.

Let Σ be a set of formulas in the variables $x_1, \ldots, x_n$. Let $\mathcal{R}$ be a model for L. We say that $\mathcal{R}$ realizes Σ if and only if some n-tuple of elements of A satisfies Σ in $\mathcal{R}$. $\mathcal{R}$ omits Σ if and only if $\mathcal{R}$ does not realize Σ. For our purposes we define a new realizability basis.

Definition 6. *Let* $\Sigma(x_1 \ldots x_n)$ *be a set of formulas of* L. *Say that a positive theory* T *in* L *positively locally realize* Σ *if and only if there is a formula* $\varphi(\mathrm{x}_1 \ldots \mathrm{x}_n)$ *in* L *s.t.*

(1) φ *is consistent with* T

(2) for all $\sigma \,\varepsilon\, \Sigma$, $\mathrm{T} \vDash \varphi$ *or* $\mathrm{T} \cup \sigma$ *is not consistent.*

Theorem 7. *Let* L_1, L_2 *be two positive languages. Let* $\mathrm{L} = \mathrm{L}_1 \cap \mathrm{L}_2$. *Suppose* T *is a complete theory in* L *and* $\mathrm{T}_1 \subset \mathrm{T}$, $\mathrm{T}_2 \subset \mathrm{T}$ *are consistent in* L_1, L_2, *respectively. Suppose there is model* M *definable from a positive diagram in the language* $\mathrm{L}_1 \cup \mathrm{L}_2$ *such that there are models* M_1 *and* M_2 *for* T_1 *and* T_2 *where* M *can be homomorphically embedded in* M_1 *and* M_2.

(i) $\mathrm{T}_1 \cup \mathrm{T}_2$ *is consistent.*

(ii) There is model N *for* $\mathrm{T}_1 \cup \mathrm{T}_2$ *definable from a positive diagram that homomorphically extend that of* M_1 *and* M_2.

Theorem 8. *Let* L_1, L_2 *be two positive languages. Let* $\mathrm{L} = \mathrm{L}_1 \cap \mathrm{L}_2$. *Suppose* T *is a complete theory in* L *and* $\mathrm{T}_1 \subset \mathrm{T}$, $\mathrm{T}_2 \subset \mathrm{T}$ *are consistent in* L_1, L_2 *respectively. Then*

(i) $\mathrm{T}_1 \cup \mathrm{T}_2$ *has a model* M, *that is positive end extension on Models* M_1 *and* M_2 *for* T_1, *and* T_2, *respectively;*

(ii) M *is definable from a positive diagram in the language* $\mathrm{L}_1 \cup \mathrm{L}_2$.

Theorem 9. *Considering a Tarskian presentation* Pup *for a theory* T *that has a positive local realization, with* T^* *we can assert the following. Every formula on the presentation* Pup *is completable in* T^*.

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The order-theoretic structure of free Heyting algebras

Michael O'Connor

A new result on the structure of Lindenbaum algebras of intuitionistic propositional logic in finitely many variables is discussed.

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Outer models, class forcing, and weakly self-defining classes

Robert Owen

M. C. Stanley described [1] a notion of class forcing that allows $0^\sharp$ to be realized from a ground model of $V = L$ assuming such models exist in the universe. We consider the Definability Lemma for such forcings and use this as a springboard to discuss set-theoretic complexity classes lying between Σ_n/Π_n and Δ_{n+1}, and their self-definability.

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Epistemic Logic and Questions

Michal Peliš

There are many formal systems in the field of logic of questions. (See nice overviews in [2] and [3].) If we omit pragmatically oriented theories, then there are two known approaches working within erotetic inferential structures: Groenendijk-Stokhof's intensional approach [1] and the general theory of erotetic inferences proposed by Andrzej Wiśniewski [4]. Even if we can meet some other theories of questions based on epistemic logic, they usually do not incorporate inferences within the language extended by questions. Mostly, questions are translated as requests for a completion of knowledge.

We would like to present some basic ideas of a non-invasive extension of epistemic logic by questions. Questions play the role of lack of an agent's knowledge. Moreover, it enables to display a structure of an agent's ignorance and a change of common knowledge in a group. Simultaneously, inferences among interrogatives and declaratives are incorporated and we can express evocation of question from a set of declaratives or implication like relationship between questions.

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Back and forth between Kripke models

Tomasz Połacik

The question whether given two structures validate the same sentences arises in a natural way in investigations of semantics of logical systems. Usually, the general answer is not obvious, if only we discard the notion of the structure isomorphism as a too restrictive one. We consider the above question in the context of Kripke semantics for intuitionistic first-order theories and introduce the notion of bounded bisimulation for first-order Kripke models.

In short, a Kripke model $\mathcal{K}$ for a given first-order language L can be viewed as a functor from a partial order $\mathbb{A}$ (viewed as a small category) to the category $\mathbb{M}$ of classical first-order structures over L whose arrows are weakly structure preserving morphism. The elements of $\mathbb{A}$ are called the nodes, and the classical structure assigned to a node α is denoted by $\mathcal{K}_\alpha$.

The bounded bisimulation for first-order Kripke models is defined as a ternary relation that may hold between two nodes α, β of Kripke models $\mathcal{K}$ and $\mathcal{M}$ respectively, and a (finite) map π from the domain of $\mathcal{K}_\alpha$ to the domain of $\mathcal{M}_\beta$. The definition involves certain back-and-forth conditions concerning both the nodes of the Kripke models, and the domains of their underlying classical structures. In particular, π will satisfy certain (finite) extension properties with respect to $\mathcal{K}_\alpha$ and $\mathcal{M}_\beta$, and structures that are related to $\mathcal{K}_\alpha$ and $\mathcal{M}_\beta$ by appropriate morphisms. We prove that if α and β are bisimilar via π to some degree c, then the domain of π satisfies at the node α exactly the same formulas of a given complexity related to c, as the image of π does at the node β. The above mentioned theorem can be viewed as a first-order Kripke model variant of the Ehrenfeucht-Fraïssé Theorem for the notion of n-partial isomorphism and the class of formulas of quantifier complexity n, viewed as the maximal number of nested quantifiers. In our theorem the complexity of a first-order formula takes into account the number of nested implications, and numbers of nested universal and existential quantifiers.

We prove basic properties of the bounded bisimulation defined above. In particular, we show that the bounded bisimulation can be described in terms of n-extendible partial isomorphism (e.g., in terms of finite Ehrenfeucht-Fraïssé games) between the appropriate classical structures of the models in question. This implies that our notion of bounded bisimulation comprises, as a particular case, the first-order bisimulation in the sense of [1]. Our results suggest how the standard techniques of classical model theory can be applied in investigation of properties of Kripke models. To illustrate this phenomena, we consider the notion of Kripke elementary submodel, and turn to the problem of constructing bounded elementary Kripke submodels of a given Kripke model. We show how this problem can be solved in the class of Kripke models over the category of ω-saturated classical structures.

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Automatic linear orders

Alexandra Revenko

A structure $\mathfrak{A} = (A, R_1, \ldots, R_n)$ is automatic if its domain A and all its relations R_i are finite automaton recognisable (automatons for relations working synchronously on tuples of finite words). The structure has an automatic presentation if it is isomorphic to some automatic structure.

It is known that there exists an algorithm that given a relation which is first order definable (with parameters) in automatic structure with an additional quantifier $\exists^\infty$ constructs an automaton recognising this relation. Hence the first order theory with this additional quantifier of an automatic structure is decidable [1, 5].

Our investigations concern the recursive isomorphism problem for two automatic presentations of linear order [3, 4].

Delhomme achieved the next result:

Theorem 3 ([2]). *An ordinal α is automatically presentable if and only if $\alpha < \omega^\omega$.*

This fact was generalized by S. Rubin and B. Khoussainov [5]. If we factor a linear order by the equivalence relation "there is a finite number of elements between x and y" then FC-rank of the linear order is a number of such factorisations after which we get a dense order or **1**-order (order with 1 element) from the given order.

Theorem 4 ([5]). *The FC-rank of every sutomatic linear order is finite.*

Thus the next results were achieved:

Theorem 5. *Every two automatic presentations of ordinal $\alpha < \omega^\omega$ are recursive isomorphic.*

The linear order is scattered if it does not contain a nontrivial dense subordering

Theorem 6. *Every two automatic presentations of automatic scattered linear order with FC-rank 2 are recursive isomorphic.*

Theorem 7. *Every automatic linear order is definable in appropriate automatic linear order with FC-rank 1.*

In addition examples of non-periodic automatic linear order were provided. The linear order is periodic if it is $\sum_{i\in\omega^*} \mathcal{A}_i + \mathcal{B} + \sum_{i\in\omega} \mathcal{C}_i$, where $\mathcal{A}_i = \mathcal{A}_j = \mathcal{A}$ for all $i \in \omega^*$, $\mathcal{C}_i = \mathcal{C}_j = \mathcal{C}$ for all $i \in \omega$ and $\mathcal{A}, \mathcal{B}, \mathcal{C}$—some linear orders

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Relevance logics and intuitionistic negation

Gemma Robles, José M. Méndez and Francisco Salto

Since the beginning of the relevance enterprise, relevance logicians have been interested in exploring the frontiers of relevance logics. This explains the considerable attention that has been paid to the paradoxical R-Mingle or the motivation behind Meyer and Routley's Classical Relevant Logic. We have investigated the result of adding the K rule to relevance logics in the context of a constructive negation and in the presence of a non-constructive negation in [4] and [1], respectively. In the present contribution, we study the effect of adding a constructive intuitionistic-type negation including the EFQ ('E falso quodlibet') axioms $A \to (\neg A \to B)$, $\neg A \to (A \to B)$ and the ECQ ('E contradictione quodlibet') axiom $(A \wedge \neg A) \to B$ to positive relevance logics up to Relevance Logic R_+ plus the mingle axiom $A \to (A \to A)$ and show that the K rule $\vdash A \Rightarrow \vdash (B \to A)$ and so, the K axiom $A \to (B \to A)$ are not derivable. Thus, a spectrum of constructive logics that, though having the standard paradoxes of consistency, lack the key paradoxes of relevance are defined.

We employ the semantics defined in [3]. Part of the results presented appear in [2].

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On cardinals in set theory without choice and regularity

Denis I. Saveliev

We show that a considerable part of standard results on cardinals can be recovered (or almost recovered) in set theory without Choice and Regularity. In the point of well-known "negative" consistency results (of Gitik and others), such "positive" provable results are rather unexpected. Further, we show that Regularity can influence on cardinals. Quite often relationships of cardinalities of well-founded sets to other cardinalities look like relationships of alephs to cardinalities of non-well-orderable sets.

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Thin projective equivalence relations and inner models

Philipp Schlicht

Greg Hjorth proved in 1993 under the assumption that all reals have sharps, that for an inner model M, M has representatives of all equivalence classes of all thin Π^1_2 equivalence relations if and only if M is Σ^1_3 correct in V and computes ω_1 correctly. I will present a similar description of the inner models that have representatives of all equivalence classes of all thin Π^1_{2n} equivalence relations, assuming PD. These are exactly the inner models that are $\Sigma^1_{(2n+1)}$-correct in V and compute the canonical tree from a $\Pi^1_{(2n-1)}$-scale correctly.

This is joint work with Greg Hjorth and Ralf Schindler.

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A modal logic of metric spaces

Mikhail Sheremet, Frank Wolter and Michael Zakharyaschev

In an attempt to extend Tarski's programme of algebraising topology to metric spaces, we introduce three binary operators on metric spaces (I, d): for $A, B \subseteq I$,

$$\begin{aligned} A \leftleftarrows B &= \{u \in I \mid d(u,A) < d(u,B)\}, \\ A \underline{\leftarrow} B &= \{u \in I \mid \forall b \in B\, \exists a \in A\, (d(u,a) < d(u,b))\}, \\ A \overset{\leftarrow}{=} B &= \{u \in I \mid \exists a \in A\, \forall b \in B\, (d(u,a) \leq d(u,b))\}, \end{aligned}$$

where $d(u, A) = \inf\{d(u, a) \mid a \in A\}$ if $A \neq \emptyset$, and $d(u, \emptyset) = +\infty$.

Denote by $\mathcal{L}$ the logic obtained by adding the operators $\leftleftarrows$, $\underline{\leftarrow}$, $\overset{\leftarrow}{=}$ to classical propositional logic and interpreting it over metric spaces (propositional variables are interpreted as their arbitrary subsets).

It is easy to see that $\mathcal{L}$ is more expressive than Tarski's $S4$; for example, $A \leftleftarrows \neg A$ is the interior of A. We show the following:

(1) $\mathcal{L}$ is as expressive over metric spaces as the logic $\mathcal{L}'$ with the operators $\exists x$ (there exists $x > 0$), $\exists^{<x}$ (in the open x-neighbourhood), and $\exists^{\leq x}$ (in the closed x-neighbourhood), where formulas starting with $\exists^{<x}$ or $\exists^{\leq x}$ are only allowed in a Boolean combination immediately after $\exists x$. For example, $A \leftleftarrows B$ is equivalent to $\exists x(\exists^{<x} A \sqcap \neg\exists^{<x} B)$.
(2) There is a natural Hilbert-style finite axiomatisation of $\mathcal{L}$.
(3) The decision problem for $\mathcal{L}$ is ExpTime-complete.
(4) Satisfiability of $\mathcal{L}$-formulas is undecidable over $\mathbb{R}$ (which is proved by reduction of the solvability problem for Diophantine equations).

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Finite Reduction Trees in Modal Logic

Tomasz Skura

A standard proof-search procedure for a propositional formula A is an attempt to construct a model for $\neg A$ by using some tableau rules. In a logic like $S4$ the resulting tree may be infinite. By placing restrictions on the application of the rules, finite procedures can be obtained. These restrictions may be regarded as *metarules*.

In this talk I present the method of finite reduction trees that are derivations in a propositional reduction system. Such a system for a logic $\mathcal{L}$ consists of *one* rule:

$$(\mathsf{R}) \qquad \frac{F}{F_1, F_2, ..., F_n}$$

satisfying the following condition:

$F \in \mathcal{L}$ *if and only if* $F_1 \in \mathcal{L}$ *or ... or* $F_n \in \mathcal{L}$.

A reduction tree for a formula A is a finite tree Υ such that:

(1) The origin of Υ is A.

(2) If $F_1, ..., F_n$ are the immediate successors of a node F, then $F_1, ..., F_n$ are obtained from F by R.

(3) The end nodes of Υ are formulas whose validity is easy to decide.

Such reduction trees can be obtained from syntactic refutation procedures (see [1]) by *deleting* the applications of modus ponens and reverse modus ponens (B/A where $\vdash A \rightarrow B$). The completeness proof is a simple inductive argument. In each inductive step we have one application of one big rule R. As a result, we can have finite reduction procedures that are justified by *proofs* that are exact, simple, and constructive.

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Provably recursive functions in extensions of a predicative arithmetic

Elliott Spoors

We provide a short summary of recent research into extensions of the weak first order theory of arithmetic $EA(I;O)$ developed by S. Wainer and previous research students, e.g., [2]. $EA(I;O)$ replaces the full induction scheme of PA by what can be seen as a predicative induction rule, incorporating the idea of variable separation from Bellantoni & Cook [1]. The principle effect is that the provably recursive functions are now the Kalmar elementary functions with the bounding functions in the ordinal analysis coming from the slow growing hierarchy.

In this talk we will present a conservative extension of $EA(I;O)$ which provides a more natural approach to composition of provably recursive functions. Furthermore, this theory can be easily extended into a hierarchy to capture higher levels of the Grzegorczyk hierarchy. The difficultly in proving so comes about in establishing upper bounds. We look in detail at the first level above our base theory where we can employ a similar finitary analysis as used on $EA(I;O)$ in [3]. In doing so we raise a number of interesting issues such as composition of the slow growing G_α functions.

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Intuitionism and repeated games

Jack D. Stecher, Harrie De Swart, and Kira Pronin

In non-cooperative game theory, a stage game is one that is played repeatedly; a standard example is an iterated prisoners' dilemma. A strategy can be part of an equilibrium of an infinitely repeated stage game, yet not part of any equilibrium if the number of stages is finite. Subjects in laboratory experiments typically play finitely repeated stage games, at least during an initial segment, as if they were playing an infinitely repeated stage game. By doing so, the subjects often receive higher payoffs than they would receive from playing equilibrium strategies.

We provide a theoretical explanation for this observed behavior by modeling the players of a game as reasoning intuitionistically. In particular, we take the position that the players of a game may not necessarily know what game they will play or what opponents they will face at stages in a sufficiently remote future. In this setting, any strategy a subject chooses in an infinite stage game is also one that the subject would be willing to choose in any stage game that had a sufficiently similar initial segment; this is essentially Brouwer's continuity principle. We show that the classically problematic behavior is intuitionistically quite sensible.

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On Ershov semilattices of degrees of Σ-definability of structures

Alexey Stukachev

The notion of Σ-definability of a structure in an admissible set, introduced by Yu. L. Ershov [1], is an effectivization of the model-theoretical notion of interpretability of structures, and, at the same time, a generalization of the notion of constructivizability of a structure on natural numbers. For structures $\mathfrak{M}$ and $\mathfrak{N}$, let $\mathfrak{M} \leqslant_\Sigma \mathfrak{N}$ means that $\mathfrak{M}$ is Σ-definable in $\mathbb{HF}(\mathfrak{N})$, the least admissible set over $\mathfrak{N}$. Preordering $\leqslant_\Sigma$, considered for structures of cardinality $\leqslant \alpha$, generates the upper semilattice $\mathcal{S}_\Sigma(\alpha)$. Σ-degrees of some uncountable structures (fields of real, p-adic and complex numbers, dense linear orders, etc.) were studied in [1, 2, 3].

We show that the semilattices of Turing and enumeration degrees are embeddable in a natural way into the semilattices of Σ-degrees, by means of Σ-degrees of structures having a degree (resp., *e*-degree). The notion of a structure having a degree, known in computable model theory, gives only a partial measure of complexity, since there are a lot of structures which do not have a degree. Σ-degrees, as well as degrees of presentability with respect to different effective reducibilities [4], are natural measures of complexity which are total, i.e., defined for any structure.

In this talk we consider some recent results on some local and global properties of Ershov semilattices [5].

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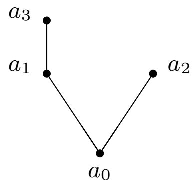
On Models Of Paraconsistent Analogue Of The Scott Logic

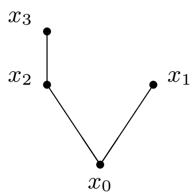
Marina Stukacheva

The Scott logic **SL**=**Li**+$\{(\neg\neg p \supset p) \supset p \vee \neg p) \supset \neg p \vee \neg\neg p\}$ (where **Li** is the intuitionistic logic) is one of the first examples of an intermediate logic with the disjunction property. In [1] it was proved that **SL**=**Li**+$X(\mu, \mathcal{D}, \perp)$, where $X(\mu, \mathcal{D}, \perp)$ is the intuitionistic canonical formula with the frame μ:

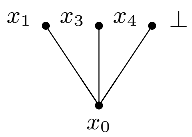


A logic is called paraconsistent if it is not intermediate and does not contain axiom $\neg p$. We study the paraconsistent analogue **Ls**=**Lj**+$\{(\neg\neg p \supset p) \supset p \vee \neg p) \supset \neg p \vee \neg\neg p\}$ of the Scott logic (**Lj** is the minimal logic).

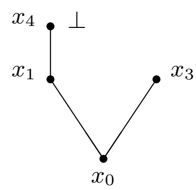
We prove that **Ls**=**Lj**+$J(\eta_1, \mathcal{D}^1)$+$J(\eta_2, \mathcal{D}^2)$+$J(\eta_3, \mathcal{D}^3)$+$J(\eta_4, \mathcal{D}^4)$+$J(\eta_5, \mathcal{D}^5)$, where $J(\eta_1, \mathcal{D}^1)$, $J(\eta_2, \mathcal{D}^2)$, $J(\eta_3, \mathcal{D}^3)$, $J(\eta_4, \mathcal{D}^4)$, $J(\eta_5, \mathcal{D}^5)$ are canonical formulas [2] with



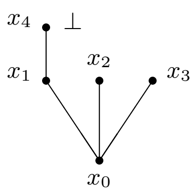
frame η_1



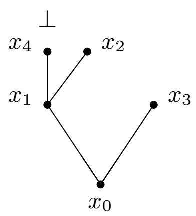
frame η_2



frame η_3



frame η_4



frame η_5

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A NEW AXIOMATIZATION OF IK_t SYSTEM

DARIUSZ SUROWIK

There are several systems of intuitionistic tense logic [1], [2], [3]. In the talk we would like to consider one of them given in [1] and called of IK_t. This is a minimal system of intuitionistic tense logic (no conditions are imposed upon earlier-later relation).

We will propose a new axiomatization for this system. Our set of axioms is smaller than the set of axioms proposed by Ewald, then the new axiomatization is simpler. We will prove our axiomatization of IK_t is equivalent to the Ewald's axiomatization.

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An analysis of some basic philosophical thesis about non-deductive inference in the framework of CHC models

Luis Adrian Urtubey

The so called CHC Models, where CHC stands for conjectures, hypothesis and consequences, have been introduced in Trillas et al. [2000] and further developed in Ying and Wang [2002] and Qiu [2007]. A CHC model is a quite general algebraic model for conjectures, hypothesis and consequences. In this model, all statements or propositions are represented as elements of an orthocomplemented lattice. Operators defined in these lattices formalize the intuitive notions of conjecture, consequence and hypothesis in an algebraic setting. Consequently, CHC models provide with a framework in which, these notions appear as mathematical objects and in which several concepts concerning issues related with conjectures, hypothesis and consequences can be properly addressed.

It is nowadays widely recognized that Charles Peirce is one of the noteworthy precursors of the logical study of non-deductive inference. Peirce coined some new terminology to designate these alternative forms of reasoning, notably the current characterization of abduction is owed to him, and he also studied some relationships among these inference patterns. This contributed talk aims at formulating some scattered philosophical thesis concerning non-deductive inference in the setting of CHC models. More specifically, the purpose of this talk is to consider, in the algebraic framework of CHC models, the representation of some facts about abduction advanced by Peirce and lately addressed by other specialists, in order to contribute to the clarification of their conceptual analysis.

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On logical theory of structures

Jacek Waldmajer

In the paper certain criteria of adequacy of representation of knowledge are presented. The criteria refer to situations in which some objects are cognized by means of other objects. The formulation of the criteria of adequacy of representation of knowledge is made within two formal-logical, axiomatic theories: *theory of cognitive tuples* **T** *and theory of structures* **TS** built over the former.

The primitive notions of T are the concepts: a *cognitive tuple* and *holding* (*occurrence*) *of a cognitive tuple*. Introduction of the notions requires adapting the following two postulates: (1) certain objects are available for cognition, (2) certain objects that are available for cognition make it possible to cognize other objects.

A *cognitive tuple* is, at the same time, intuitively understood as a tuple of objects by means of which we want to cognize something. If objects of these tuples are connected with one another, so that information (knowledge) of the cognized objects is transferred by their means, then we can say about the cognitive tuples that they *hold* (*occur*), and their description is called *adequate representation of knowledge*.

Since cognition of a given object often occurs by means of more than one cognitive tuple, representation of knowledge usually refers to a certain set of cognitive tuples. This leads to taking into account, in theoretical considerations, a notion of *structure*--indispensable in scientific research [1]. This concept is introduced in the theory **TS**. **TS** describes these properties of the structures which point out some criteria of adequacy of representation of knowledge (cf. [2]). An important notion of this theory is the one of *adequate structure*. Such a structure is characterized by that all of its cognitive tuples are holding (occurring) tuples, and the knowledge, which is transferred by them is *adequate*.

The theory **TS** has its interpretation in *ZF* set theory and as such is consistent.

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Harmonious many-valued propositional logics and the logic of computer networks

Heinrich Wansing and Yaroslav Shramko

In many-valued logic, sometimes a distinction is made not only between designated and undesignated (not designated) truth values, but between designated, undesignated, and antidesignated truth values. But even if the set of truth values is, in fact, tripartitioned, usually only a single semantic consequence relation is defined that preserves the possession of a designated value from the premises to the conclusions of an inference. We shall argue that if the set of anti-designated values does not constitute the complement of the set of designated values, it is natural to define *two* entailment relations, a positive one that preserves possessing a designated value from the premises to the conclusions of an inference, and a negative one that preserves possessing an antidesignated value from the conclusions to the premises. Once this distinction has been drawn, it is quite natural to reflect it in the logical object language and to contemplate many-valued logics whose language is split into a positive and a matching negative logical vocabulary. If the positive and the negative entailment relations do not coincide, the interpretations of matching pairs of connectives are distinct, and nevertheless the positive entailment relation restricted to the positive vocabulary is isomorphic to the negative entailment relation restricted to the negative vocabulary, then such a many-valued logic is called *harmonious*. We shall present examples of harmonious finitely-valued logics. These examples are not *ad hoc*, but emerge naturally in the context of generalizing Nuel Belnap's ideas on how a single computer should think to how interconnected computers should reason.

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Automatic proof generation in Kleene algebra with tests

James Worthington

Kleene algebra (KA) is the algebra of regular events. Familiar examples of Kleene algebras include regular sets, relation algebras, and trace algebras. A Kleene algebra with tests (KAT) is a Kleene algebra with an embedded Boolean subalgebra. The addition tests allows one to encode while programs as KAT terms, thus the equational theory of KAT allows one to reason about (propositional) program equivalence. More complicated statements about programs can be expressed in the Hoare theory of KAT, which suffices to encode Propositional Hoare Logic.

In our paper, we prove the following results. First, there is a PSPACE transducer which takes equations of Kleene Algebra as input and outputs proofs of them in an algebraic proof system based on a form of bisimulation. Second, we give a feasible reduction from the equational theory of KAT to the equational theory of KA. Combined with the fact that the Hoare theory of KAT reduces efficiently to the equational theory of KAT, this yields an algorithm capable of generating proofs of a large class of statements about programs.

Our result has applications to areas such as Proof-Carrying Code, where it is necessary that a formal proof be produced. There are two traditional approaches to the equational theory of KA: interactive protocols for generating proofs and the decision procedure of Stockmeyer and Meyer to determine finite automata equivalence. Each has its own drawbacks: interactive protocols are by definition not automatic, and the output of a decision procedure is just one bit, and therefore not efficiently verifiable. Our method, which is completely automatic and outputs a proof, combines the best features of each.

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On vague set and vague logic

Urszula Wybraniec-Skardowska and Zbigniew Bonikowski

The subject matter of the consideration touches the problem of vagueness from the logical, set-theoretical and the computer science perspective. The paper proposes a new, formal-logical approach to the problem.

The starting point is the concept of agent's unit information about any object o of discovered reality with respect to a relation R defined on U. It is the image $\overrightarrow{R}(o)$ of the object o wrt R. If $\overrightarrow{R}(o)$ is unknown to the agent, then the unit information for him can be given as the equation:

$$(e) \qquad \overrightarrow{R}(o) = X,$$

where X is an unknown quantity. Its scope is the at least a two-element family $\mathbf{V}_o$ of sets (relations) that are possible solutions of (e) from the point of view of the agent. Then we say that the agent's unit information about o wrt R is *vague* and the family $\mathbf{V}_o$ is a *vague set*.

A language representation of (e) is the vague sentence:

$$(re) \qquad a \text{ is } V \text{ or } V(a),$$

where a is the singular term of o and V is the name-predicate (resp. vague-predicate) corresponding to X; the denotation of V is the vague set $\mathbf{V}_o$ – the family of all denotations (extensions) of sharp terms representing V from the agent's point of view.The upper and lower approximations (limits) of $\mathbf{V}_o$ are algebraic boundaries in $P(U)$. Some operations on vague sets and their algebraic properties are also presented. Some important conditions about the membership relation for vague sets, in connection to Zadeh's fuzzy sets (1965), Pawlak's rough sets (1982) and Blizard's multisets (1989) are established as well. A view on the problem of logic of vague sentences (*vague logic*) based on vague sets is also discussed. The considerations intend to take into account a 'conservative', classical approach to reasoning based on vague premises.

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CM-triviality and geometric elimination of imaginaries

Ikuo Yoneda

CM-triviality is a geometric notion of forking introduced by Hrushovski in [3]. The original definition is given in terms of eq-structures. Usually the CM-triviality of a generic relational structure is shown as follows; first show the weak elimination of imaginaries of the structure, and then working in the real sort we complete the proof.

In my talk, I present a much more direct way: I define the notion of *CM-triviality in the real sort which implies* both the geometric elimination of imaginaries and the CM-triviality in the original sense.

I also give a sufficient condition for the geometric elimination of imaginaries in simple theories and (pregeometric) surgical theories.

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Fragments of Martin's axiom related to the rectangle refining property

Teruyuki Yorioka

A partition $[\omega_1]^2 = K_0 \cup K_1$ has the rectangle refining property if for any $I, J \in [\omega_1]^{\aleph_1}$, there are $I' \in [I]^{\aleph_1}$ and $J' \in [J]^{\aleph_1}$ such that for every $\alpha \in I'$ and $\beta \in J'$ with $\alpha < \beta$, $\{\alpha, \beta\} \in K_0$. This property has been defined by Larson-Todorčević to solve Katětov's problem.

In 1980's, Stevo Todorčević has studied fragments of Martin's axiom. Let $\mathsf{MA}_{\aleph_1}$ be Martin's axiom for $\aleph_1$-many dense sets of ccc forcing notions, $\mathcal{K}_2$ the statement that every ccc forcing notion has the property K, $\mathcal{C}^2$ the statement that any product of ccc forcing notions still has the ccc. We note that $\mathsf{MA}_{\aleph_1}$ implies $\mathcal{K}_2$, and $\mathcal{K}_2$ implies $\mathcal{C}^2$. However it has been unknown whether these reverse implications hold.

In this talk, we consider new chain condition of forcing notions. A forcing notion $\mathbb{P}$ has the anti-rectangle refining property if it is uncountable and for any $I, J \in [\mathbb{P}]^{\aleph_1}$, there are $I' \in [I]^{\aleph_1}$ and $J' \in [J]^{\aleph_1}$ such that for every $p \in I'$ and $q \in J'$, p and q are incompatible in $\mathbb{P}$. Let $a(\mathbb{P})$ be the forcing notion adding an antichain in $\mathbb{P}$ by finite approximations. If a forcing notion $\mathbb{P}$ has the anti-rectangle refining property, then for any $I, J \in [a(\mathbb{P})]^{\aleph_1}$ with $I \cup J$ pairwise disjoint, there are $I' \in [I]^{\aleph_1}$ and $J' \in [J]^{\aleph_1}$ such that for every $\sigma \in I'$ and $\tau \in J'$, σ and τ are compatible in $a(\mathbb{P})$, that is, $\sigma \cup \tau \in a(\mathbb{P})$. This property is stronger than the countable chain condition. Let $\mathsf{MA}_{\aleph_1}(a(\text{arec}))$ be the $\mathsf{MA}_{\aleph_1}$ for all forcing notions $a(\mathbb{P})$ such that $\mathbb{P}$ has the anti-rectangle refining property.

We can show that it is consistent that $\mathsf{MA}_{\aleph_1}(a(\text{arec}))$ holds but $\mathcal{C}^2$ fails, etc.

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‘Asymmetric’ Systems Of Natural Deduction

Ryan Young

The construction of natural deduction systems normally requires the basic symmetry that both elimination and introduction rules are given for each connective within the system. There are many good reasons for this requirement, however it is not a necessary requirement for the construction of a logic. The project of this paper is to explore the consequences of relaxing this restriction in a particular direction, namely removing $\bigvee$-*Introduction*.

The motivation for this investigation is the observation that uniquely among the standard classical inference rules, $\bigvee$-*Introduction* seems out of place in ordinary human argumentation. One does not normally argue from a singular premise, say “It will rain tomorrow”, to the conclusion that “It will rain tomorrow or my watch will break tomorrow”. Unless there is some known connection between it raining, and the state of my watch, such an argument would be considered wrong in ordinary discourse.

Given that Natural Deduction most closely resembles human reasoning, and human reasoning normally proceeds without the $\bigvee$-*Introduction* rule, it should be worth investigating Natural Deduction systems without the $\bigvee$-*Introduction* rule. In this paper, one such system will be the focus: Intuitionistic Natural Deduction without $\bigvee$-*Introduction*. This system suffers from the limitations that are to be expected when one removes the basic symmetry of normal Natural Deduction systems. It is, for example, not complete in any normal sense, and there is no obvious model for it. Nevertheless, it has a number of unusual properties that are worth exploring. For example, one can prove consistency directly from the inference rules; it collapses to Classical Logic upon the addition of the Law of the Excluded Middle as an axiom; and it is ‘negatively explosive’. That is, from a contradiction the negation of every formula is provable, but not every formula. The paper concludes with a brief discussion of the semantic interpretation of this system, and the significance of the formal results.

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On the Second Order Intuitionistic Propositional Logic Without Universal Quantifier

Konrad Zdanowski

We examine the second order intuitionistic propositional logic, IPC2. Let $\mathcal{F}_\exists$ be the set of formulas with only existential quantification. We prove Glivenko's theorem for the formulas in $\mathcal{F}_\exists$ that is, for $\varphi \in \mathcal{F}_\exists$, φ is a classical tautology if and only if $\neg\neg\varphi$ is a tautology of IPC2. We show that for each sentence $\varphi \in \mathcal{F}_\exists$ (without free variables), φ is a classical tautology if and only if φ is an intuitionistic tautology. As a corollary we obtain a semantical argument that the quantifier $\forall$ is not definable in IPC2 from $\bot, \vee, \wedge, \rightarrow, \exists$.

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Abstracts Presented by Title

An interpretation about space and time in quantum mechanics

Adib Ben Jebara

There was a repeated experiment where at first, two protons are joined and of opposite spins. Then, the second is taken far away, and it is acted upon the first to modify its spin. The second proton will change its spin to keep it the opposite of the spin of the first (change at a speed greater than the speed of light). For further details: `http://mist.npl.washington.edu/npl/int_rep/tiqm/TI_24.html#2.4.1`

Now, if you will assume with me that we can apply the set theory ZFU to physical space, U (urelements) being physical space, you will see that we get an interpretation of the experiment.

Indeed, as it is not possible to define a distance in U, the second proton will not be any more far away from the first.

Also, if we consider time to be U, we cannot say that the protons were separated a long time ago and that there should be no more influence.

Such two hypothesis about space and time were already made in “About time and time of elementary particles” in ASL Annual Meeting 2005. It is not yet clear: are space and time alternatively U? Is U space-time?

Continuums are still approximations.

Mr. Andreas Blass pointed out that, in the experiment, the first proton is acted upon for measurement, and he also pointed out that only the most used distances are not defined in U. He is skeptical about the assumptions.

As the hypothesis apply to cosmology, the unity of physics would be increased in such a direction.

There was another repeated experiment with a photon, expected to go one way, going both two quite separated ways. Here, again, if we assume something else about space, the two ways would be not that much separated.

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Aristotle's many-sorted logic

John Corcoran

As noted in 1962 by Timothy Smiley, if Aristotle's logic is faithfully translated into modern symbolic logic, the fit is exact. If categorical sentences are translated into many-sorted logic *MSL* according to Smiley's method or the two other methods presented here, an argument with arbitrarily many premises is valid according to Aristotle's system if and only if its translation is valid according to modern standard many-sorted logic. As William Parry observed in 1973, this result can be proved using my 1972 proof of the completeness of Aristotle's syllogistic.

MSL Using Sortal Variables

The ranges of the portal variables are all non-empty. In the examples, ess ranges over spheres, pee over polygons.

Every sphere is a polygon.	$\forall s \exists p\ s = p$
Some sphere is a polygon.	$\exists s \exists p\ s = p$
No sphere is a polygon.	$\forall s \forall p\ s \neq p$
Some sphere isn't a polygon.	$\exists s \forall p\ s \neq p$

MSL Using Range-Indicators with General (*Non-Sortal*) *Variables*

Each initial variable occurrence follows an occurrence of a quantified range-indicator or 11common noun" that determines the range of the variable in each of its occurrences in the quantifier's scope. To each range-indicator, a non-empty set is assigned as its "extension". In the example, the extension of ess is the spheres, pee the polygons.

For every sphere x, there exists a polygon y such that $x = y$.	$\forall \mathrm{S}x \exists \mathrm{P}y\ x = y$
For some sphere x, there exists a polygon y such that $x = y$.	$\exists \mathrm{S}x \exists \mathrm{P}y\ x = y$
For every sphere x, for every polygon y, x isn't p.	$\forall \mathrm{S}x \forall \mathrm{P}y\ s \neq y$
There exists a sphere x such that, for every polygon y, x isn't y.	$\exists \mathrm{S}x \forall \mathrm{P}y\ x \neq y$

Many-sorted logic with range-indicators and non-sortal variables was pioneered by Anil Gupta in his 1980 book. Also see my *Logical form of quantifier phrases: quantifier-sortal-variable* this BULLETIN, vol. 5 (1999) pp. 418–419.

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Axiomatizations for intersections of substructural logics

Nikolaos Galatos

Substructural logics include classical, intuitionistic, linear, relevant and many-valued logics, and their study provides a framework for the comparative study of various logics. Substructural logics are defined as axiomatic extensions of the sequent calculus FL; the latter is essentially obtained from Gentzen's system LJ for intuitionistic propositional logic, by omitting the structural rules of exchange, contraction and weakening. Residuated lattices form algebraic semantics for substructural logics and algebraic methods have been applied successfully to the logical investigation. Substructural logics ordered under inclusion, form a lattice. Given axiomatizations for two substructural logics, their join in the lattice (the least logic containing both of them) is axiomatized by the union of the axiomatizations. On the other hand, the meet of the two logics (the greatest logic containing both of them) is their intersection, but the intersection of their axiomatizations does not axiomatize the meet; for example, it can be empty. Using algebraic methods, we provide axiomatizations for intersections of substructural logics that also work in case the language lacks the disjunction connective. In particular, we prove that the intersection of two finitely axiomatized subcommutative substructural logics has a finite axiomatization, as well. En route, we investigate the finitely subdirectly irreducible algebras of residuated (semi)lattices (if join is not included in the language) and show that if they form an elementary class, the finitely axiomatized extensions of the corresponding substructural logic are closed under intersections.

This is joint work with J. G. Raftery and J. S. Olson.

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Estimation of algorithmic complexity of computable models classes

Eugene N. Pavlovsky

In recent research of Goncharov and Knight [1] there was given a classification of of computable models classes with given theoretical-model properties on structural and antistructural.

In this paper I considered such a natural classes as: ω-categorical models, ω_1-categorical, prime, homogenous, saturated models, models with strongly-minimal theories. Models are considered in predicate computable language. I gave precise, upper and lower estimations of complexity classes:

There are:

1) class of finite models is Σ^0_2-complete.

2) classes of models with ω-categorical theories, prime models, homogenous models lies in $[\Sigma^0_\omega,\Pi^0_{\omega+2}]$ complexity interval.

3) class of models with ω_1-categorical theories is in $[\Sigma^0_\omega,\Sigma^0_{\omega+1}]$ complexity interval.

4) class of saturated models is Σ^1_1-complete.

The proofs contain a new approach of achieving lower bounds of $\varnothing^{(\omega)}$ complexity degrees, which uses Goncharov-Marker modified construction lowering complexity degree of model saving some model-theoretical properties [2].

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An explicit basis for admissible rules of modal logics extending S4.1

V. V. Rematski

Let a modal logic $\lambda \supseteq S4.1$ has (i) Finite Model Property; (ii) a weak co-cover property (WCCP for short); (iii) the disjunction property be (cf. [2]).

We introduce the following sequence of inference rules:

$$\mathcal{R}_n := \frac{\Box(A_{n,1} \wedge \neg(A_n \wedge B)) \vee \Box z}{\Box\neg A_n \vee \Box z};$$

where $A_n := \bigwedge_{1\leq i\leq n} \Diamond p_i$; $A_{n,1} := \Box[\bigwedge_{1\leq i\leq n} p_i \rightarrow \neg\Box q]$; $B := q \vee \neg\Box q$; $n > 1, n \in N$

Theorem 0.1. *Let modal logic $\lambda \supseteq S4.1$ have: (i) FMP; (ii) WCCP; (iii) disjunction property. Then rules $\mathcal{R}_n$, $n \in N$, form a basis for λ-admissible rules.*

Conclusion 0.2. *Rules $\mathcal{R}_n$, $n \in N$, form a basis for all admissible rules of logics S4.1, Grz.*

Theorem 0.3. *Let modal logic $\lambda \supseteq Grz$ have: (i) FMP; (ii) $\mathcal{L}$WCCP (iii) disjunction property; (iv) the width of λ is equal to finite $\mathcal{L}$. Then rules $\mathcal{R}_n$, $1 < n \leq \mathcal{L}$, form basis for all λ-admissible rules.*

Theorem 0.4. Let logic $\lambda \supseteq$ *Int* satisfies to conditions (i)–(iii) of theorem 0.1 or (i)–(iv) of theorem 0.3 Then rules $\{\mathcal{R}^i_n : T(\mathcal{R}^i_n) = \mathcal{R}_n, n > 1, n \in N\}$ form a basis for λ-admissible rules. If the width of λ is equal finite to $\mathcal{L}$ then a basis for λ-admissible rules is $\{\mathcal{R}^i_n : T(\mathcal{R}^i_n) = \mathcal{R}_n, 1 < n \leq \mathcal{L}\}$.

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Discrete linear temporal logic with current time point clusters

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Diverse variations of linear temporal logics, as special bi-modal logics, were studied very profoundly. Though, even nowadays there are not investigated pathways. Only recently [1] it has been shown that all finitely axiomatizable linear tense logics are decidable and coNP-complete. We study temporal logic $\mathcal{TL}(N_C, \Box_w^{+-})$ based on linear time with current time point clusters (bi-modal clusters situated in place of natural numbers imitating time flow). Its language uses, together with standard modalities $\diamond^+$ (possible in future) and $\diamond^-$ (possible in past), special temporal operations, $-\Box_w^+$ (weakly necessary in future) and $\Box_w^-$ (weakly necessary in past). Continuing [1], we first show that the logic $\mathcal{TL}(N_C, \Box_w^{+-})$ itself is decidable. We propose an deciding algorithm based on reduction of formulas to rules and converting rules in special reduced normal form, and (then) on checking validity of such rules in models of single exponential size in reduced forms. Then we show how to reduce in $\mathcal{TL}(N_C, \Box_w^{+-})$ the admissibility problem of inference rules to the decidability of $\mathcal{TL}(N_C, \Box_w^{+-})$ itself. So, we prove that the admissibility problem for $\mathcal{TL}(N_C, \Box_w^{+-})$ is also decidable. This fact (based on checking validity of inference rules and presence of a definable universal modality) extends previous results [2, 3].

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The first level predicate calculus of mutually-inversistic logic is quantifier-free

Xunwei Zhou

The first level predicate calculus of mutually-inversistic logic is quantifier-free. A proposition in the form of parent$(x, y) \wedge$ ancestor$(y, z) \leqslant^{-1}$ ancestor(x, z) is called a first-order single empirical or mathematical connection proposition, where x, y, z are term variables (corresponding to individual variables in classical logic), $\wedge$ is a fact compounder, $\leqslant^{-1}$ (mutually inverse implication) is an empirical or mathematical connective. The boundness of a term variable in a first-order single empirical or mathematical connection proposition is determined by its occurrences relative to empirical or mathematical connective, fact compounder, predicate, and function. A term variable is relevantly bound if it occurs on both sides of the empirical or mathematical connective, such as the x in man$(x) \leqslant^{-1}$ mortal(x). A term variable is transitively bound if it occurs on one side of the empirical or mathematical connective but on both sides of a fact compounder, such as the y in parent$(x, y) \wedge$ ancestor$(y, z) \leqslant^{-1}$ ancestor(x, z). A term variable is additionally bound if it occurs on one side of the empirical or mathematical connective but on both sides of a binary predicate, such as the z in the elimination law of addition $x + z = y + z \leqslant^{-1} x = y$. A term variable is juxtaposed bound if it occurs on one side of a predicate but on both sides of a binary function such as the z in $x * x + y * y + z * z \leqslant 1 \leqslant^{-1} x * x + y * y \leqslant 1$.

If a term variable occurs only once in a first-order single empirical or mathematical connection proposition, then it is free, Because it cannot be one of the above-mentioned bound variables. If a term variable occurs in a first-order single empirical or mathematical connection proposition just twice, then it is purely bound. Because it is just one of the above-mentioned boundness. For example, in parent$(x, y) \wedge$ ancestor$(y, z) \leqslant^{-1}$ ancestor(x, z), x and z are purely relevantly bound, y is purely transitively bound. If a term variable occurs in a first-order single empirical or mathematical connection proposition three times or more, then it is combined boundness. Because it combines two or more of the above-mentioned boundness. For example, x in $x < y \wedge x < z \leqslant^{-1} x < y + z$ is combined relevant and transitive boundness. Because the first and second occurrences of x are transitively bound, the first and third occurrences of x are relevantly bound, the second and the third occurrences are relevantly bound.

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