THE AUTOMORPHISM TOWER OF CENTERLESS GROUP WITHOUT CHOICE

ITAY KAPLAN, SAHARON SHELAH

Given a centerless group one can embed it into its automorphism group which is again centerless. One can define a tower of groups, G^α, such that $G^{\alpha+1} = \mathrm{Aut}(G^\alpha)$. The Automorphism tower problem asks when does this construction stabilizes, i.e. when does $G^\alpha = G^{\alpha+1}$. Simon Thomas gave a short and simple set theoretic proof that it stops after less than $(2^{|G|})^+$ steps. His proof, though simple, uses the axiom of Choice quite strongly. A couple of years ago, we succeeded in giving a Choiceless proof, which was also more informative. Recently, we have managed to improve that proof so that now it gives even more information, and covers some cases that we were not covered before by our proof (but followed easily from Thomas' proof). The sort of information this proof gives may have some implication on the still open problem of finding a bound for the tower when the group is countable, using Polish spaces and descriptive set theory.