Survey on the first homology groups of strong types in model theory

Junguk Lee

Abstract

J. Goodrick, B. Kim, and A. Kolesnkiov gave a framework for a homology theory of strong types in model theory, which is related with the generalized amalgamation properties. In stable theory, they showed that for $n \geq 1$, if a strong type p has k-amalgamation for $k \leq n+1$, then the n-th homology group is trivial if and only if the $(n+2)$-amalgamation in p fails. For the case $n = 1$, that is, for the first homology groups, it was not known much in general. In my talk, we aim to give a concrete relation between the first homology groups and the Lascar groups in arbitrary theory. For this, we mainly concern the following topics:

- A classification of 2-chains with 1-shell boundaries.
- Canonical projection map from the Lascar group to the first homology group of a strong type.
- Description of the first homology group of a strong type.

As applications, we get equivalent conditions for a strong type to be a Lascar type and for the Lascar group to be trivial in terms of the first homology group.