Ellis semigroup: a Boolean approach

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Abstract

We consider a group G acting by left translation on its space of types $S(G)$. Thus $S(G)$ is a G-flow. The dynamical properties of this G-flow are explained by its Ellis semigroup $E(S(G))$, which is just the topological closure of the semi-group of homeomorphisms of $S(G)$ induced by elements of G, in the space of all functions $G \to G$ (with the topology of pointwise convergence).

The minimal flows in $E(S(G))$ are just the minimal left ideals $I \lhd G$. Each such an ideal splits into a disjoint union of groups of the form uI, where u is an idempotent. All such subgroups of $E(S(G))$ are isomorphic.

In the talk I will explain the nature of these groups by interpreting the Ellis semigroup as a semigroup of automorphisms of a certain Boolean algebra of (externally) definable subsets of G. Then the fact that all groups of the form uI are isomorphic will become clear.

This interpretation of the Ellis semigroup enables us also to deal with the absoluteness questions, that is the questions, how the dynamical properties of tthe flow $S(G)$ change, if we replace G by its elementary extension.