ON MODELS OF PARACONSISTENT ANALOGUE OF THE SCOTT LOGIC

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We consider the class JHN of all non-trivial extensions of the minimal (or Johans-son's) logic (denoted L_j).
It is well-known that JHN partition into three disjoint subclasses [Odintsov 1998].
There are

the class INT of intermediate logics,

the class NEG of negative logics containing formula $\perp$, and

the class PAR of proper paraconsistent extensions of L_j consisting of logics not belonging to the first two classes.

In 1989 Zakharyashchev associated with every finite rooted intuitionistic frame $\mu = \langle W, R \rangle$ a formula such that it is refuted in a frame iff this frame contains a subframe reducible to μ and satisfies some other natural conditions.

M. V. Zakharyashchev [1989], *Syntax and semantics of intermediate logics*, Algebra and Logic, vol.28, pp. 402-429.

In 2005 we defined the canonical formulas for extensions of the minimal logic.
In this case we obtained a necessary and sufficient condition for the refutability of the minimal canonical formulas in general frames and proved completeness theorem for that canonical formulas.

M. Stukacheva [2006] On canonical formulas for extensions of the minimal logic // Siberian Electronic Mathematical Reports. – 2006. – Vol. 3. – P. 312–334.
`http:// semr.math.nsc.ru`

Definition (Segerberg, 1968)

We call Kripke j-frame, or simply j-frame, a triple $\mu=\langle W, R, Q\rangle$, where $\langle W, R\rangle$ is an ordinary Kripke frame for intuitionistic logic and $Q \subseteq W$ is a cone (upward closed set) with respect to R, which we will call the cone of abnormal worlds.

Worlds lying out of Q are called *normal*.

Note that a valuation v of a j-frame μ, a model $\mathcal{M}=\langle\mu, v\rangle$ and the forcing relation $\models$ between models and formulas are defined in just the same way as for ordinary Kripke frame. The only exception is in the case of constant $\perp$:

$$\mathcal{M} \models_x \perp \Longleftrightarrow x \in Q.$$

K. Segerberg [1968] *Propositional logics related to Heyting's and Johansson's*, Theoria, vol.34, pp.26-61.

Definition

A general frame for JHN is a pair $\mathfrak{M} = \langle \mu, S \rangle$*, where* $\mu = \langle W, R, Q \rangle$ *is a j-frame and S is a collection of upward closed subsets of W which contains* $\emptyset$*, Q and is closed under* $\cap$*,* $\cup$*, and operation* $\supset$ *is defined as follows :*

$\forall X, Y \in S \; X \supset Y = \{x \in W | \forall y \in W (xRy \text{ and } y \in X) \Rightarrow y \in Y)\}$.

If S contains all upward closed subsets of W, then $\mathfrak{M} = \langle \mu, S \rangle$ is in effect an ordinary Kripke j-frame.

Let $\mathfrak{M} = \langle W, R, Q, S \rangle$ be a finite rooted general j-frame with $e_0, \ldots, e_n$ being all the distinct elements of W and e_0 the root.

Definition

A pair δ=$(\overline{x}, \overline{y})$ of nonempty sets $\overline{x}$, $\overline{y} \subseteq W$ is called a d-domain in $\mathfrak{M}$ if the following conditions are satisfied:

1. *$\overline{x}$ and $\overline{y}$ are antichains in W, and $\overline{x}$ has at least two elements;*
2. $(\forall x \in \overline{x})(\forall y \in \overline{y})(\neg xRy)$;
3. $(\forall z \in W)(\forall x \in \overline{x}\ (zRx) \Rightarrow \exists y \in \overline{y}\ (zRy))$.

Let $\mathcal{D}$ be some (possibly empty) set of d-domains in $\mathfrak{M}$.
By $\mathcal{D}_1$ we denote the set of all d-domains such that $\overline{y} \cap (W \setminus Q) \neq \emptyset$ and by $\mathcal{D}_2$ – the set of all d-domains with property $\overline{y} \subseteq Q$.
With $\mathfrak{M}=\langle \mu, S \rangle$ and $\mathcal{D}$ we associate the following *JHN-canonical formula*

$$J(\mu, \mathcal{D}) \rightleftharpoons (\bigwedge_{e_i R e_j} A_{ij}) \wedge (\bigwedge_{\delta \in \mathcal{D}} B_\delta) \wedge C \supset p_0,$$

where

$$C = \bigwedge_{i=1}^{m} (\wedge \Gamma_i \supset p_i \vee \bot) \supset \bot;$$

$$\Gamma_j = \{p_k \mid \neg e_j R e_k\};$$

and

- if $\delta = (\overline{x}, \overline{y}) \in \mathcal{D}_1$, then

  B_δ=$\bigwedge_{e_i \in \overline{y}, e_i \notin Q}(\wedge\Gamma_i \supset p_i \vee \bot) \wedge \bigwedge_{e_i \in \overline{y}, e_i \in Q}(\wedge\Gamma_i \wedge \bot \supset p_i) \supset \bigvee_{e_j \in \overline{x}} p_j$,

  (besides it, if $\overline{y} \cap Q$=$\emptyset$, then there is no second conjunctive term);
- if $\delta = (\overline{x}, \overline{y}) \in \mathcal{D}_2$, then

  B_δ=$\bigwedge_{e_i \in \overline{y}}(\wedge\Gamma_i \wedge \bot \supset p_i) \supset \bigvee_{e_j \in \overline{x}} p_j$;

a term A_{ij} define as follows:

- if $e_i \notin Q$, $e_j \notin Q$, then $A_{ij} \rightleftharpoons (\wedge\Gamma_j \supset p_j \vee \perp) \supset p_i$;
- if $e_i \notin Q$, $e_j \in Q$, then $A_{ij} \rightleftharpoons (\wedge\Gamma_j \wedge \perp \supset p_j) \supset p_i$;
- if $e_i \in Q$, $e_j \in Q$, then $A_{ij} \rightleftharpoons (\wedge\Gamma_j \supset p_j) \supset p_i$.

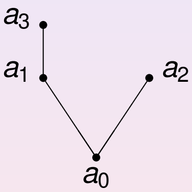
The Scott logic

$$\mathbf{SL}{=}\mathbf{Li}{+}\{(\neg\neg p \supset p) \supset p \vee \neg p) \supset \neg p \vee \neg\neg p\}$$

(where **Li** is the intuitionistic logic and **Li**=**Ij** $+ \{\perp \supset \mathbf{p}\}$)

is one of the first examples of an intermediate logic with the disjunction property.

Zakharyashchev proved that **SL**=**Li**$+X(\mu, \mathcal{D}, \bot)$, where $X(\mu, \mathcal{D}, \bot)$ is the intuitionistic canonical formula with the frame μ:



We study the paraconsistent analogue

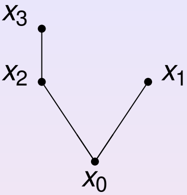
$$\mathbf{Ls}{=}\mathbf{Lj}+\{(\neg\neg p \supset p) \supset p \vee \neg p) \supset \neg p \vee \neg\neg p\}$$

of the Scott logic.

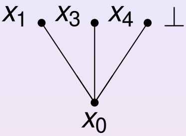
Theorem

Ls=**Lj**$+J(\eta_1, \mathcal{D}^1)+J(\eta_2, \mathcal{D}^2)+J(\eta_3, \mathcal{D}^3)+J(\eta_4, \mathcal{D}^4)+J(\eta_5, \mathcal{D}^5)$,

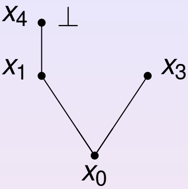
where $J(\eta_1, \mathcal{D}^1)$, $J(\eta_2, \mathcal{D}^2)$, $J(\eta_3, \mathcal{D}^3)$, $J(\eta_4, \mathcal{D}^4)$, $J(\eta_5, \mathcal{D}^5)$ *are canonical formulas [2] with*



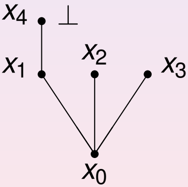
frame η_1



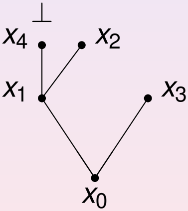
frame η_2



frame η_3



frame η_4



frame η_5