

Epistemic Logic with Questions

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Questions as a part of inferential structures

Inferential Erotetic Logic (A. Wiśniewski, based on classical logic)

Evocation

$$\langle \Gamma, Q \rangle$$

Erotetic implication

$$\langle \langle Q_1, \Gamma \rangle, Q_2 \rangle$$

Example of e-implication

Q_1 : What is Peter graduate of: faculty of law or faculty of economy?

I can be satisfied by the answer

He is a lawyer.

even if I did not ask

Q_2 : What is Peter: lawyer or economist?

The connection between Q_1 and Q_2 could be done by the following knowledge base Γ :

Someone is graduate of a faculty of law iff he/she is a lawyer.

Someone is graduate of a faculty of economy iff he/she is an economist.

One-agent propositional epistemic logic

propositional language with modality K (knowledge as “necessity”) and M ($M\varphi \equiv \neg K\neg\varphi$)

semantics

- **Kripke frame** $\mathcal{F} = \langle S, R \rangle$ with a set of states (points, indices, possible worlds) S and an accessibility relation $R \subseteq S^2$.
- **Kripke model** $\mathbb{M} = \langle \mathcal{F}, \models \rangle$ where \models is a satisfaction relation between states and formulas.

The satisfaction relation \models is defined by a standard way:

1. For each $\varphi \in \mathcal{A}$ and (M, s) : either $(M, s) \models \varphi$ or $(M, s) \not\models \varphi$.
2. $(M, s) \models \neg\varphi$ iff $(M, s) \not\models \varphi$
3. $(M, s) \models \psi_1 \vee \psi_2$ iff $(M, s) \models \psi_1$ or $(M, s) \models \psi_2$
4. $(M, s) \models \psi_1 \wedge \psi_2$ iff $(M, s) \models \psi_1$ and $(M, s) \models \psi_2$
5. $(M, s) \models \psi_1 \rightarrow \psi_2$ iff $(M, s) \models \psi_1$ implies $(M, s) \models \psi_2$
6. $(M, s) \models K\varphi$ iff $(M, s_1) \models \varphi$, for each s_1 such that sRs_1

Incorporating questions

extend epistemic language by ? and appropriate brackets

$$Q =? \underbrace{\{\alpha_1, \dots, \alpha_n\}}_{dQ}$$

Q requires one of the following answers:

It is the case that α_1 .

:

It is the case that α_n .

A questioner presupposes at least $(\alpha_1 \vee \dots \vee \alpha_n)$ and maybe more.

Presuppositions

presupposition of a question Q

$\varphi \in \text{Pres}Q$ iff $(\forall \mathbf{M})(\forall s)(\forall \alpha \in dQ)((\mathbf{M}, s) \models \alpha \rightarrow \varphi)$

prospective presupposition of a question Q

$\varphi \in \text{PPres}Q$ iff $\varphi \in \text{Pres}Q$ and $(\forall \mathbf{M})(\forall s)$

$(\mathbf{M}, s) \models \varphi$ implies $(\exists \alpha \in dQ)((\mathbf{M}, s) \models \alpha)$

- Each prospective presupposition is a maximal presupposition.
- If $\varphi, \psi \in \text{PPres}Q$, then $\varphi \equiv \psi$.

Q is sound at (\mathbf{M}, s)

$$(\mathbf{M}, s) \models Q$$

iff

1. $(\forall \alpha \in dQ)$

(a) $(\mathbf{M}, s) \models M\alpha$

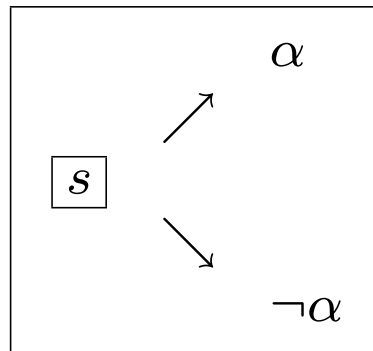
(b) $(\mathbf{M}, s) \not\models K\alpha$

2. $(\forall \varphi \in \text{Pres}Q)((\mathbf{M}, s) \models K\varphi)$

A question sound at (\mathbf{M}, s) forms a partitioning on the afterset.

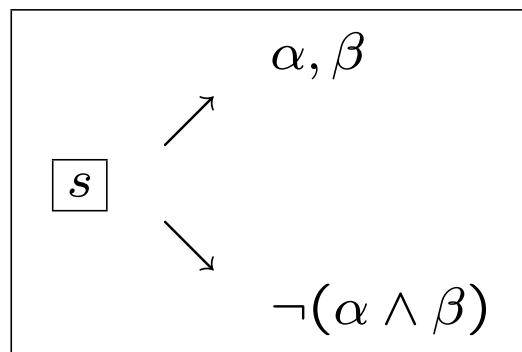
Examples 1

- $(M, s) \models ?\alpha$ means



The same is for $(M, s) \models ?\neg\alpha$.

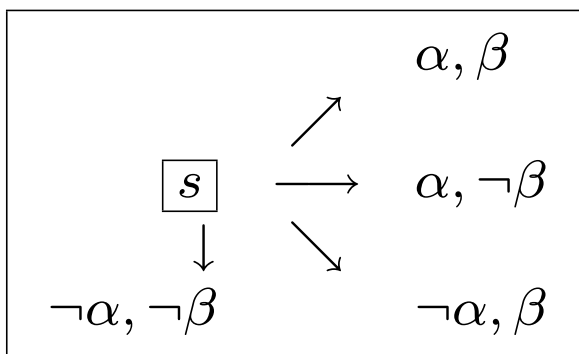
- $(M, s) \models ?(\alpha \wedge \beta)$



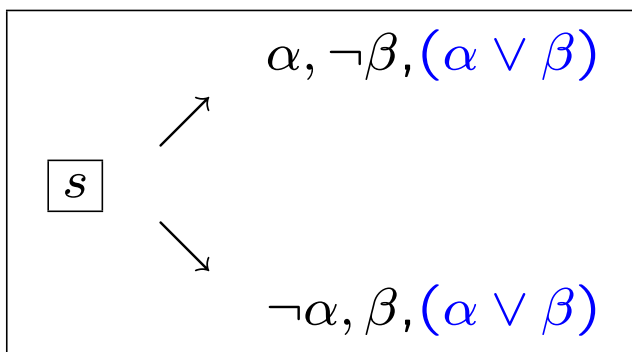
Analogously for $?(\alpha \vee \beta)$.

Examples 2

- $|\alpha, \beta|$ is equal to $\{(\alpha \wedge \beta), (\neg\alpha \wedge \beta), (\alpha \wedge \neg\beta), (\neg\alpha \wedge \neg\beta)\}$.



- $(M, s) \models \{\alpha, \beta\}$, then $(M, s) \models K(\alpha \vee \beta)$



Evocation

$$(\mathbf{M}, s) \models \Gamma \xrightarrow{i} Q$$

iff

$$(\mathbf{M}, s) \models K\Gamma$$

and

$$(\mathbf{M}, s) \models Q$$

coincides with *question in an information set*
(J. Groenendijk, M. Stokhof)

E-implication

$$(\mathbf{M}, s) \models (\Gamma, Q_1) \Rightarrow Q_2$$

iff

$$((\mathbf{M}, s) \models K\Gamma \text{ and } (\mathbf{M}, s) \models Q_1)$$

implies

$$(\mathbf{M}, s) \models Q_2$$

Pure e-implication ($\Gamma = \emptyset$)

$$(\mathbf{M}, s) \models Q_1 \rightarrow Q_2$$

iff

$$(\mathbf{M}, s) \models Q_1 \text{ implies } (\mathbf{M}, s) \models Q_2$$

Examples of pure e-implication

- $\models ?\alpha \rightarrow ?\neg\alpha$ as well as $\models ?\alpha \leftarrow ?\neg\alpha$
- $\models ?(\alpha \wedge \beta) \leftarrow ?|\alpha, \beta|$, the same for \vee instead of \wedge
- $\models ?|\alpha, \beta| \rightarrow ?\alpha$ and $\models ?|\alpha, \beta| \rightarrow ?\beta$
- $\models ?|\alpha, \beta| \rightarrow ?(\alpha|\beta)$
- $\models ?|\alpha, \beta| \rightarrow ?\{\alpha, \beta, (\neg\alpha \wedge \neg\beta)\}$
- $\models ?\{(\alpha \vee \beta), \alpha\} \rightarrow ?\{\alpha, \beta\}$
- $\models ?\{\alpha, \beta, \gamma\} \rightarrow ?\alpha$ (as well as $?\beta$ and $?\gamma$)

Example of e-implication

$$\Gamma = \{(\alpha_1 \leftrightarrow \beta_1), (\alpha_2 \leftrightarrow \beta_2)\}$$

$$\models (\Gamma, ?\{\beta_1, \beta_2\}) \Rightarrow ?\{\alpha_1, \alpha_2\}$$

as well as

$$\models (\Gamma, ?\{\alpha_1, \alpha_2\}) \Rightarrow ?\{\beta_1, \beta_2\}$$

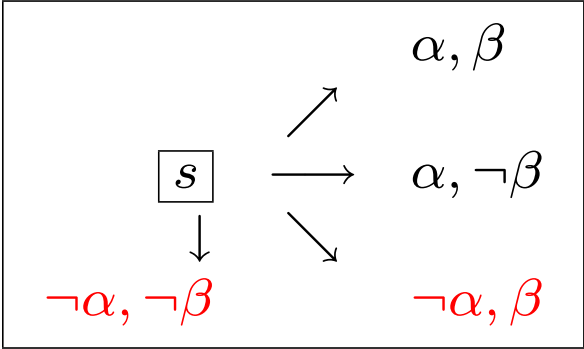
both questions are equal with respect to Γ

Answerhood

An agent *gets* a **complete answer** φ to a question Q at (M, s) iff $(M, s) \models K\varphi$ such that $\varphi \models \alpha$ for some $\alpha \in dQ$.

An agent *gets* a **partial answer** to a question Q at (M, s) iff she gets a complete answer to a question $? \varphi$ at (M, s) such that $Q \rightarrow ? \varphi$.

α is a partial answer to $?|\alpha, \beta|$



Basic references

D. Harrah. The logic of questions. In D. Gabbay and F. Guentner (eds.), *Handbook of Philosophical Logic*. Kluwer, 2002. Volume 8, pages 1–60.

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J. Groenendijk and M. Stokhof. Questions. In J. van Benthem and A. ter Meulen (eds.), *Handbook of Logic and Language*. Elsevier, 1997. Pages 1055–1125.