Intuitionism and Repeated Games

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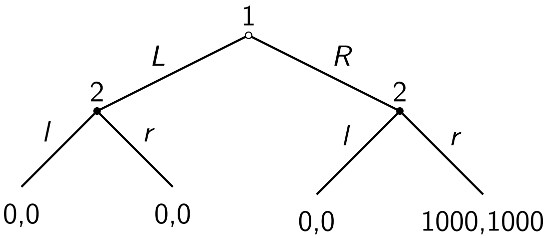

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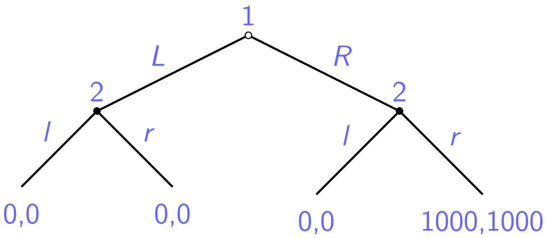

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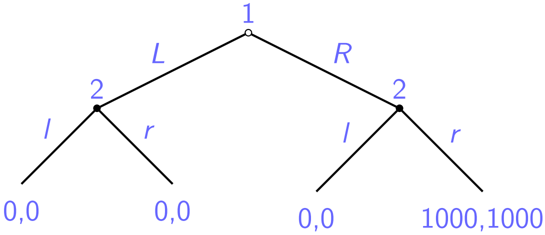

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- Selten: (L, l, l) is only equilibrium because player 2 plans to do something crazy on a node he never reaches.

Backward Induction

- In the game we saw above, it is common to use a backward induction argument to rule out the strange equilibria.
- Intuition: restrict attention to cases where everyone plays optimally, even at nodes that are never reached.
- Start with the terminal nodes and work backwards up the tree.
- In the last game we saw, this rules out all equilibria except those where the observed path is (R, r).

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Example

- Even numbered rounds: Player 1 can end the game and get $n/2+1$; Player 2 gets $n/2-1$ in this case. Or Player 1 can continue the game
- Odd numbered rounds: Player 2 can end the game and get $(n+1)/2$: Player 1 gets $(n-1)/2-1$ in this case. Or Player 2 can continue the game.
- Game must end no later than round 100.
- Payoffs if ending in rounds 0, 1, 2, and 3 are $(1,-1);(-1,1);(2,0);(0,2)$.

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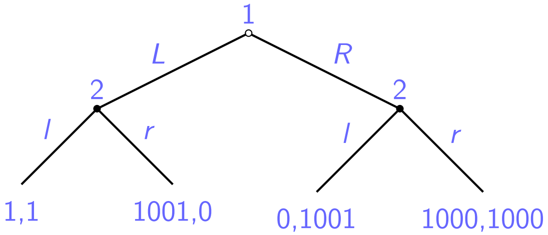

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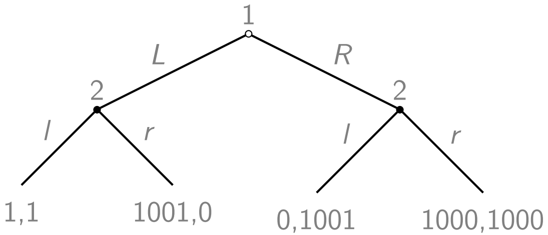
- One might also think of a prisoners' dilemma repeated for $10^{10^{10}}$ rounds:
- The backward induction argument seems extreme. And for the most part, people don't play games this way.
- Other approaches to refinements of Nash equilibria rely on continuity properties (typically on other players' strategies). We choose to go in this direction.

Infinite Games

- First observation: the situation changes dramatically if the game is an infinite-horizon game.
  - Caveat: We need to be careful about what we mean by the players' payoffs if the game is infinite.
  - Common approaches: long-run average, discounted sums (i.e., multiply payoff in round n by δ^n for some $\delta \in (0, 1)$, etc.
- In finitely-repeated games, discounting doesn't help. But in infinitely-repeated games, the players can enforce collusion:



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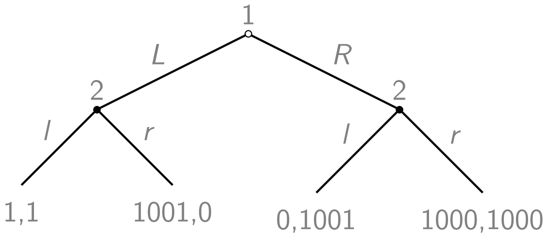

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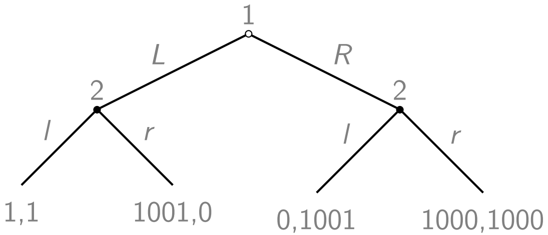

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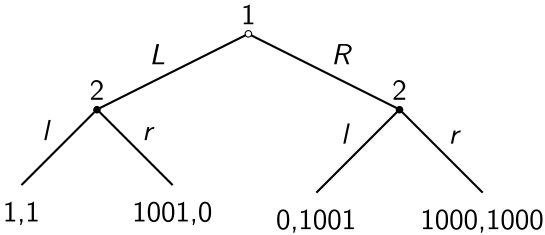

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- We start by considering the class of all games played in (countably) infinitely many stages.
- Games played for finitely-many stages are really just a special case. (Put a trivial game in every round after the last round.)
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Alternative Approach (cont'd)

- Main idea: the infinite stage games are a set of sequences (chioce sequences if we include partially specified games).
- Each prefix of a sequence defines a collection of infinitely-repeated games—the set of all possible continuations.
- This is the base of a topology.
  - The longer the initial segment on which two stage games are identical, the closer they are to each other in this topology.
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- Every fully-specified infinite game is an isolated point. And the set of such games is of the first Baire category.
- The continuity requirement essentially says that anyone who will play cooperatively in an infinitely-repeated game must be willing to do so for some time in a finitely-repeated game.
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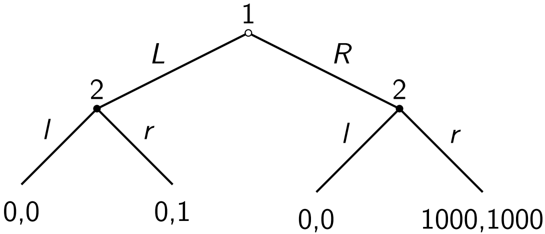

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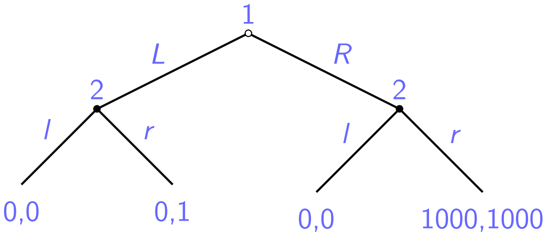

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