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WEAK LIMITS AND INTEGRALS OF GAUSSIAN COVARIANCES IN BANACH SPACES

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Abstract: Let E be a separable real Banach space not containing an isomorphic copy of c_0. Let $\mathcal{S}$ be a subset of $\mathcal{L}(E^*, E)$ with the property that each $Q \in \mathcal{S}$ is the covariance of the centred Gaussian measure μ_Q on E. We show that the weak operator closure of $\mathcal{S}$ consists of Gaussian covariances again, provided that

$$\sup_{Q \in \mathcal{S}} \int_E \|x\|^2 d\mu_Q(x) < \infty.$$

If in addition E has type 2, the same conclusion holds for the weak operator closure of the convex hull of $\mathcal{S}$. As an application, sufficient conditions are obtained for the integral of Gaussian covariance operators to be a Gaussian covariance. Analogues of these results are given for the class of γ-radonifying operators from a separable real Hilbert space H into E.

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